Comprehensive Assessment of the Depth-Averaged Richard’s Equation (DARE) model

Junhao He¹, Latif Kalin¹, Mohamed M. Hantush², Sabahattin Isik¹, Mehdi Rezaeianzadeh³

¹ Department of Forestry and Wildlife Sciences, Auburn, AL
² U.S. EPA National Risk Management Research Laboratory, Cincinnati, OH
³ NOAA Affiliate, NOAA National Water Center, Tuscaloosa, AL
The results and conclusions in this presentation are solely those of the authors and have not been formally reviewed by the U.S. EPA and should not be construed to represent any agency determination or policy.
DARE converts partial differential equations of RE into ordinary equations.

DARE has good performance for estimating average soil moisture.

Comprehensive assessment of DARE is necessary to understand its advantages and limitations.
Model Assessment

Scenarios:

- Effect of soil layer thickness
- Layered soil with sand and clay on top of each other
- Different soil characteristics models
- Different numerical methods
- Sensitivity analysis
- Application to real data

Results of 1, 2, 3 and 5 were compared with HYDRUS-1D model as reference
HYDRUS-1D

- HYDRUS-1D is a finite element model for simulating the one-dimensional movement of water and solute in variably saturated media.
- Average moisture content from HYDRUS:

\[
\bar{\theta}_1 = \frac{1}{N_1} \sum_{i=1}^{N_1} \theta_i \\
\bar{\theta}_2 = \frac{1}{N_2} \sum_{i=N_1+1}^{N_2} \theta_i
\]
Model Assessment and Results

1. Effect of soil layer thickness

Soil textures (uniform): Sand, Loam, Silt-clay-loam

Layer depth:

A
L1=10 cm
L2=10 cm

B
L1=10 cm
L2=30 cm

C
L1=10 cm
L2=60 cm

D
L1=30 cm
L2=30 cm

E
L1=30 cm
L2=60 cm

Top BC: no rain no ET

Bottom BCs:
1. Free-drainage
2. Variable flux

Initial condition: 80% saturation
Model Assessment and Results

Free-drainage BC:

**Sand**
- $L_1=10, L_2=10$
- $L_1=10, L_2=30$
- $L_1=10, L_2=60$
- $L_1=30, L_2=30$
- $L_1=30, L_2=60$

**Loam**
- $L_1=10, L_2=10$
- $L_1=10, L_2=30$
- $L_1=10, L_2=60$
- $L_1=30, L_2=30$
- $L_1=30, L_2=60$

**Silt-Clay-Loam**
- $L_1=10, L_2=10$
- $L_1=10, L_2=30$
- $L_1=10, L_2=60$
- $L_1=30, L_2=30$
- $L_1=30, L_2=60$
Variable flux BC:

- **Sand**
  - L1=10, L2=10
  - L1=10, L2=30
  - L1=10, L2=60

- **Loam**
  - L1=10, L2=10
  - L1=10, L2=30
  - L1=30, L2=30
  - L1=30, L2=60

- **Silt-Clay-Loam**
  - L1=10, L2=10
  - L1=10, L2=30
  - L1=10, L2=60

Model Assessment and Results
Model Assessment and Results

2. Fine vs coarse textured soil layers

Soil profiles:

A

Clay

Sand

B

Sand

Clay

Scenarios:

1. \( i = 0 \text{ cm/day}, \ T = 0 \text{ cm/day} \)
2. \( i = 0.5 \text{ cm/day}, \ T = 0.2 \text{ cm/day} \)

Bottom BC: water table at the bottom

Initial condition: 80% saturation

L1=10 cm, L2=20 cm
L1=30 cm, L2=60 cm
# Model Assessment and Results

## Clay/Sand:

### Depth

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 cm</td>
<td>10 cm</td>
</tr>
<tr>
<td>30 cm</td>
<td>30 cm</td>
</tr>
</tbody>
</table>

### Top BCs

<table>
<thead>
<tr>
<th>Depth</th>
<th>i=0 cm/day, T=0 cm/day</th>
<th>i=0.5 cm/day, T=0.2 cm/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>0.35</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>0.4</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
</tbody>
</table>

- L1: Length of Layer 1
- L2: Length of Layer 2

- DARE_L1
- DARE_L2
- HYDRUS_L1
- HYDRUS_L2
Model Assessment and Results

Sand/Clay

Top BCs
Depth

L1=10 cm
L2=10 cm

\( i=0 \text{ cm/day, } T=0 \text{ cm/day} \)

\( i=0.5 \text{ cm/day, } T=0.2 \text{ cm/day} \)

HYDRUS crushed

L1=30 cm
L2=30 cm

HYDRUS crushed

HYDRUS crushed
Brooks and Corey (1964) model:

\[ S_e = \begin{cases} |\alpha h|^{-n} & \psi < 0 \\ 1 & \psi \geq 0 \end{cases} \]

\[ K(S_e) = K_s S_e^{\frac{2}{n}+1}+2 \]
4. Numerical methods

Runge-Kutta method:

\[ \tilde{\theta}_{i+1} = \tilde{\theta}_i + \frac{\Delta t}{6} \left( f(t_i, q_i) + f(t_{i+0.5}, q_{i+0.5}) + f(t_{i+1}, q_{i+1}) \right) \]

\[ + f(t_{i+0.5}, q_{i+0.5}) + f(t_{i+1}, q_{i+1}) \]
Model Assessment and Results

5. Sensitivity analysis

Two-way Kolmogorov–Smirnov (K-S) test
Monte-Carlo simulations

- Generate input parameters
- 10,000 simulations
- Run models
- Calculate RMSE
- Identify behavior (B) and non-behavior (B') sets
- Plot CDF of B and B'
- Calculate $D_{\text{max}}$ value

<table>
<thead>
<tr>
<th></th>
<th>L1</th>
<th>L2</th>
<th>θr1</th>
<th>θr_L2</th>
<th>θs_L1</th>
<th>θs_L2</th>
<th>Ks_L1</th>
<th>Ks_L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1</td>
<td>1</td>
<td>0.034</td>
<td>0.034</td>
<td>0.36</td>
<td>0.36</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Max</td>
<td>90</td>
<td>90</td>
<td>0.1</td>
<td>0.1</td>
<td>0.46</td>
<td>0.46</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>α_L1</th>
<th>α_L2</th>
<th>n_L1</th>
<th>n_L2</th>
<th>λ_L1</th>
<th>λ_L2</th>
<th>θi_L1</th>
<th>θi_L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.001</td>
<td>0.001</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Max</td>
<td>0.2</td>
<td>0.2</td>
<td>3</td>
<td>3</td>
<td>1.5</td>
<td>1.5</td>
<td>0.36</td>
<td>0.36</td>
</tr>
</tbody>
</table>
Model Assessment and Results

Free-Drainage BC:

- **θs_L1**
- **θs_L2**
- **n_L1**
- **n_L2**
- **θi_L1**
- **θi_L2**

* indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001
Model Assessment and Results

Variable flux BC

- $\theta_s\_L1$
- $\theta_s\_L2$
- $\text{Depth}\_L1$
- $\text{Depth}\_L2$
- $\alpha\_L1$
- $\alpha\_L2$

* indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001
Site level soil moisture data and meteorology data were collected from Tuskegee, AL monitored by Soil Climate Analysis Network (SCAN). Daily soil moisture data were collected at five soil depth (5, 10, 20, 50 and 100 cm). Data were gathered from Jan 1st 2018 to March 31st 2019.

Meteorology data includes daily high, mean and low temperature, rainfall intensity, wind speed, relative humidity etc. Potential evapotranspiration was calculated from meteorology data by Penman-Monteith method.
Model Assessment and Results

Model calibration

- Data was split into two parts for calibration and validation.
- Data from Feb 1\textsuperscript{st} 2018 to Jun 1\textsuperscript{st} 2018 was used for model calibration.
- Measured soil moisture was calculated by averaging moisture content from sensors within each layer.
Model Assessment and Results

Model validation

- Data from Feb 1st 2019 to May 31st 2019 was used for model validation.
Conclusion

• $\theta_i$ and $n$ are very sensitive parameters for free-drainage BC. Layer depth and $\alpha$ are most sensitive parameters for variable flux BC.

• DARE is a stable and efficient solution of RE for modeling unsaturated water movement in a two-layered soil.

• For large scale applications or in field-scale models where only average moisture content of few soil layers are needed, DARE can be a robust methodology for simulating soil moisture dynamics.

Pros

1. DARE can deal with soil layers with different characteristics
2. The choice of the numerical method has no effect on DARE soil moisture estimates
3. The CPU time required to run DARE were about 3 to 4 times less than that for HYDRUS

Cons

1. DARE does not work as well with fine textured soils
2. Performance decreases with thicker layer depth
3. DARE shows more anomalies than HYDRUS when using the Brooks and Corey model
Thank You

Any questions?