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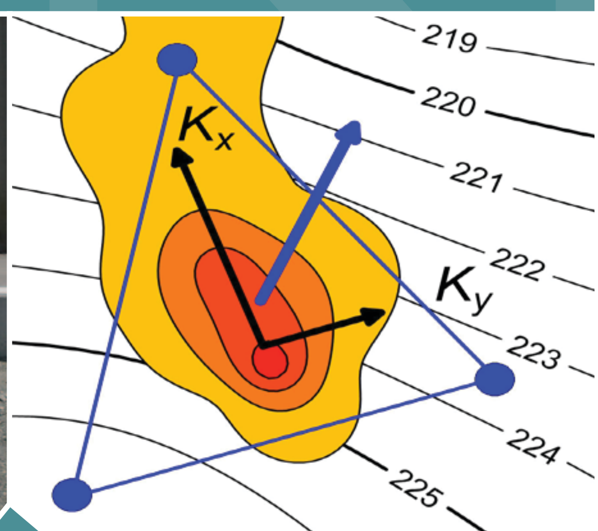
3PE: A Tool for Estimating Groundwater Flow Vectors



Office of Research and Development
National Risk Management Research Laboratory | Ground Water and Ecosystems Restoration Division

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3PE: A Tool for Estimating Groundwater Flow Vectors

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This document has been reviewed in accordance with U.S. Environmental Protection Agency policy and approved for publication.

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Abstract

Evaluation of hydraulic gradients and the associated groundwater flow rates and directions is a fundamental aspect of hydrogeologic characterization. Many methods, ranging in complexity from simple three-point solution techniques to complex numerical models of groundwater flow, are available for hydraulic gradient estimation. In many situations where the water table or other potentiometric surface can be represented as a plane, three-point estimation methods will provide a quick and cost-effective means for estimating hydraulic gradients, particularly for initial evaluation purposes.

The three-point solution method is well suited for implementation in a spreadsheet format. 3PE is an interactive spreadsheet developed in Microsoft Excel® for estimation of horizontal hydraulic gradients and groundwater velocities. It is particularly well suited for analyzing transient hydrologic conditions, allowing rapid visualization of hydraulic gradient and groundwater velocity vectors. Applications include groundwater remediation performance assessments, hydrologic conceptualization of groundwater/surface water interactions, and general characterization of site-specific hydrology. Site-specific investigation objectives, supported by an analysis of measurement uncertainty, provide the framework for determining the most appropriate measurement strategies and monitoring network designs for estimating hydraulic gradients.

1.0 Introduction

Evaluation of groundwater flow directions and rates is a fundamental component of most hydrogeologic investigations. At sites with groundwater contamination, information concerning spatial and temporal groundwater flow patterns is used to identify potential receptors, design remediation systems, and evaluate the effectiveness of existing systems. The determination of how the groundwater flow field changes in response to natural or man-made stresses often drives the development of the site-specific conceptual model in ecological as well as groundwater contamination investigations.

Hydraulic gradients and, therefore, groundwater flow direction and magnitude may vary with time in response to factors such as precipitation, pumping, irrigation, or interactions with surface water bodies. During groundwater remediation, hydraulic stresses such as remedial fluid injection and/or groundwater extraction can result in significant changes to the groundwater flow field. Physical modifications to aquifer structure such as installation of permeable reactive barriers or low permeability barriers may also significantly alter the groundwater flow field. Characterization of the aquifer response to such changes is a challenging aspect of the evaluation of remedial effectiveness.

In simple terms, groundwater flow is controlled by the hydraulic conductivity of the aquifer materials and hydraulic gradients. Hydraulic gradient is the change in hydraulic head per unit of distance measured in the direction of the maximum rate of decrease in hydraulic head, and represents the slope of the water table or other potentiometric surface. The hydraulic gradient, a vector having both a direction and magnitude, is the driving force for groundwater flow within an aquifer. Groundwater seepage velocity within an aquifer is often estimated using a form of Darcy's Law, estimated hydraulic gradients, and estimates of hydraulic conductivity and effective porosity.

Typical tools for calculating hydraulic gradients range from simple graphical solutions (e.g., Heath, 1983; Fetter, 1981) using hydraulic head (i.e., groundwater elevation) measurements from three locations forming a triangle (i.e., three-point problem) to complex three-dimensional models of groundwater flow. In situations where key areas within the groundwater flow field can be represented by a plane or a series of planes, calculation of hydraulic gradients using a mathematical three-point solution method offers a relatively simple and rapid means for evaluating spatial and temporal changes in the magnitude and direction of groundwater flow. Groundwater flow can often be adequately represented by a plane in portions of the aquifer separating areas of aquifer recharge from discharge areas. Depending on site-specific hydrology, areas with planar groundwater flow often occur at the site to sub-regional scales and can be indicated on water table or other potentiometric surface maps as areas where equipotential lines are straight and evenly spaced.

In many situations, three-point solution methods can be particularly useful when combined with data acquisition using data logging pressure transducers for automated measurement of groundwater elevations. The spreadsheet 3PE is a tool developed using an extension of the three-point problem approach to allow rapid calculation of the horizontal direction and magnitude of groundwater flow in horizontally isotropic and anisotropic settings. The 3PE workbook represents an extension of previously available spreadsheet tools such as those developed by Kelly and Bogardi (1989) and Devlin (2003) and on-line calculators such as the Three Point Gradient calculator (USEPA, 2014). The 3PE spreadsheet was developed in Microsoft Excel® workbook format using a rigorous mathematical approach based on vector analysis, matrix algebra, and trigonometric functions. Required user inputs include well coordinates defining a triangle of wells (i.e., a three-point problem); estimates of hydraulic conductivity and effective porosity of the aquifer materials, if groundwater velocity is to be calculated; the direction in which the hydraulic conductivity is greatest, if the aquifer has horizontal anisotropy; and the date/time and hydraulic head values for each of the three wells. 3PE calculates hydraulic gradients and groundwater velocities for each set of hydraulic head measurements. The spreadsheet format is ideally suited as both a teaching tool (e.g., for examining the effects of changes in input parameters such as hydraulic conductivity) and for rapid visualization and evaluation of seasonal and other temporal fluctuations in groundwater flow rates and directions. This tool is anticipated to be used by project personnel with technical backgrounds, especially by hydrogeologists, to evaluate the site-specific spatial and temporal changes in hydraulic gradients and associated groundwater velocities.

Potential applications for three-point analysis methods and the 3PE workbook include:

- Evaluation of the hydraulic impact of groundwater remediation systems.
- Conceptualization of potential groundwater/surface water interactions.
- Rapid visualization of spatial and temporal patterns in hydraulic gradients and groundwater flow directions.
- Enhanced groundwater flow model calibration through improved characterization of the range of hydrologic conditions.
- Improved conceptualization of site hydrology.

The theoretical development and derivation of the mathematical equations solved in 3PE are provided in Appendix A. Comparisons of the results of hydraulic gradient and groundwater velocity calculations using 3PE with results from published problems and other published calculators are presented in Appendix B. The 3PE workbook with an example data set is provided in Appendix C.

2.0 Description of the Spreadsheet Tool

The 3PE spreadsheet was developed using standard functions built into Microsoft Excel®. There are no macros or user-defined functions in the spreadsheet, increasing portability. It is designed to estimate the hydraulic gradient vector (the direction and magnitude) from the coordinates of three wells and hydraulic head measurements. Data can be entered manually or copied from pressure transducer or other files. In the case where data from more than one triangle of wells are available, a separate workbook for each triangle of wells would be used.

The tool allows calculation of horizontal hydraulic gradient and groundwater velocity vectors in settings where horizontal hydraulic conductivity is either isotropic or anisotropic. A geologic formation is isotropic if the properties of the medium (e.g., hydraulic conductivity) are independent of direction. If the properties vary with the direction of the measurement, the medium is anisotropic. In anisotropic aquifers, specially designed aquifer tests are used to estimate the direction and magnitude of the axes of maximum and minimum hydraulic conductivity. In some cases, a heterogeneous aquifer may be effectively represented using anisotropic conditions when it can be conceptualized as a homogeneous, anisotropic aquifer. Examples include a fractured aquifer that can be represented as an equivalent porous medium with higher hydraulic conductivity in the direction of dominant fracture orientation and some coal bed aquifers (e.g., Stoner, 1981; Kern and Dobson, 1998). In the case of a horizontally isotropic aquifer, the groundwater flow direction coincides with the hydraulic gradient. However, in an anisotropic aquifer, the groundwater flow direction is not necessarily parallel to the hydraulic gradient vector (Figure 1). The groundwater flow direction will depend on the orientation and magnitudes of the maximum and minimum hydraulic conductivity and the orientation of the hydraulic gradient.

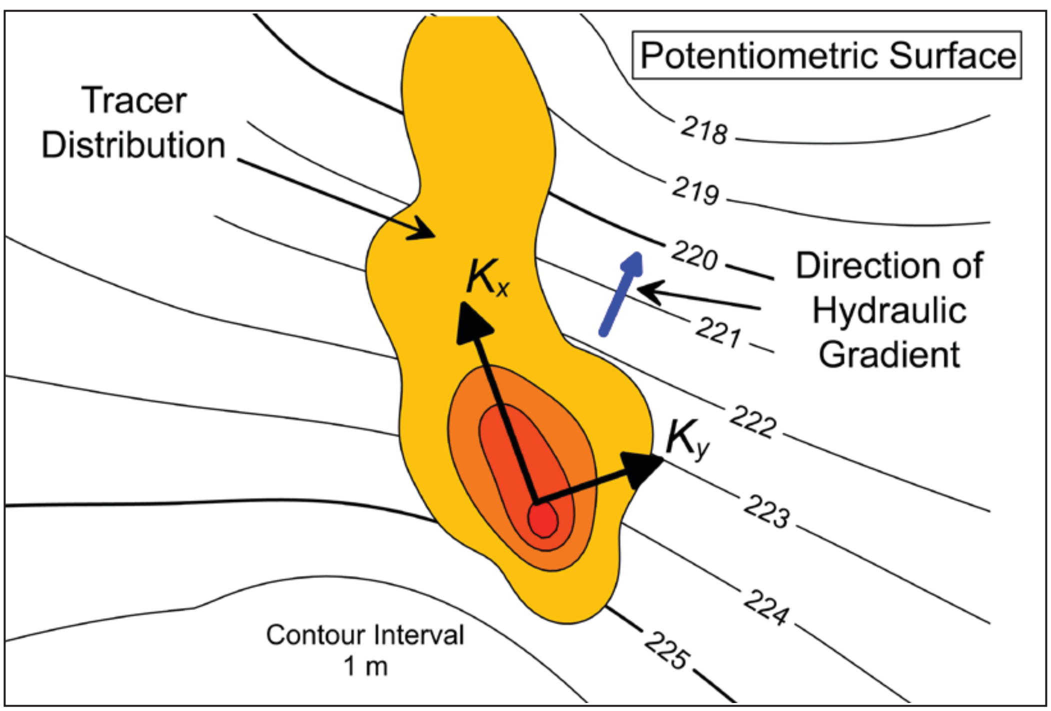

Figure 1. In an anisotropic setting, the direction of the hydraulic gradient may be different from the direction of groundwater flow. The direction in which hydraulic conductivity is highest (K_{max}) is illustrated by the axis labeled K_x. The axis labeled K_y illustrates the direction in which hydraulic conductivity is lowest (K_{min}). For this example, the magnitude of K_{max} is much greater than K_{min}. The resulting groundwater flow direction is evidenced by the elongated distribution of the dissolved tracer compound. This direction is essentially parallel to K_x and significantly different from the hydraulic gradient direction.

Estimates of the magnitude of the groundwater velocity vector (i.e., rate of groundwater flow) can also be calculated by 3PE using Darcy's Law if estimates of the hydraulic conductivity and the effective porosity of the aquifer materials are provided. In general, such estimates should be considered preliminary and only local in nature since geologic settings often cannot be adequately represented using a single value of hydraulic conductivity.

The spreadsheet can be used to evaluate the gradient and velocity using a single triangle with one or many sets of hydraulic head measurements. The spreadsheet calculates and records the coordinates of the starting and ending points of each hydraulic gradient and velocity vector. The vectors are plotted within the workbook to allow rapid inspection. The length of the vector in the plots represents its magnitude. This allows the user to visually inspect not only the direction of the hydraulic gradient or groundwater flow but, also, to observe changes in the magnitudes of the vectors with time. In that sense, the spreadsheet is ideal for analyzing transient hydrologic conditions. The spreadsheet results can also be copied to other software for production of customized graphics such as rose diagrams displaying the distribution of hydraulic gradients or groundwater flow directions, histograms for evaluation of vector directions or magnitudes, and for overlays on site-specific base maps or other geographical information systems outputs.

2.1 Input Data

All cells in the 3PE spreadsheet for user input of data are shaded green. Cells that are shaded blue present significant results of the hydraulic gradient and velocity calculations. Any consistent system of length and time units can be used in 3PE (e.g., feet and days, meters and days). However, the same set of units must be used in all inputs.

Header Information

Space is provided at the top of the 3PE spreadsheet for user input of project-specific text information such as the site name, location, and measurement dates (Figure 2). Any text may be entered in these cells.

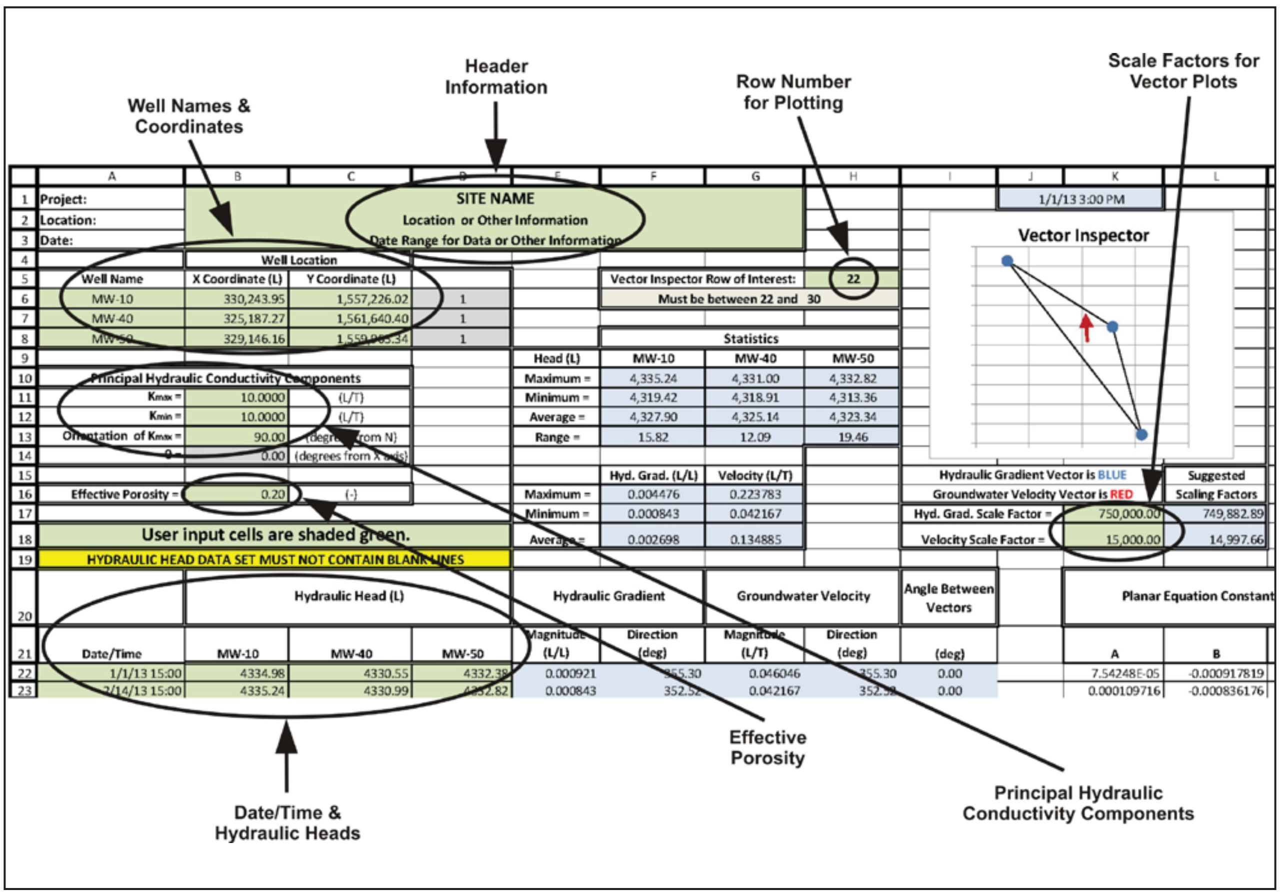

Figure 2. Screen capture from the 3PE spreadsheet depicting data to be input by user (green cells). Blue cells are significant results of the calculations.

Well Names and Coordinates

The spreadsheet requires the name and location in two-dimensional Cartesian coordinates of each well forming the triangle of the three-point problem: Well Name, X Coordinate, and Y Coordinate (Figure 2). Any consistent set of length units may be used for the well coordinates.

Principal Hydraulic Conductivity Components and Effective Porosity

For conditions of horizontal anisotropy, the user must enter the maximum (K_{max}) and minimum (K_{min}) horizontal hydraulic conductivity values (Figure 2) and the orientation of the axis of maximum hydraulic conductivity (K_{max}) in degrees measured clockwise from North. **Although calculated values for the orientations of the hydraulic gradient and groundwater velocity vectors can range from 0 to 360 degrees, the orientation of the axis of maximum horizontal hydraulic conductivity must be input as a value between 0 and 180 degrees.** This direction and representative values for the magnitude of the maximum and minimum horizontal hydraulic conductivity will generally be obtained using specialized hydraulic tests (e.g., Hantush, 1966; Way and McKee, 1982; Neuman et al., 1984; Maslia and Randolf, 1987). It is assumed that K_{max} is orthogonal to K_{min}.

In the more common case of an aquifer that is represented as isotropic in the horizontal plane, hydraulic conductivity is uniform (i.e., $K_{max} = K_{min}$). Thus, the same value will be entered for both K_{max} and K_{min} and the orientation is ignored in the 3PE calculations. The length units used in specifying hydraulic conductivity must be consistent with those used in specifying well coordinates and hydraulic head.

Effective porosity is entered as a fraction between zero and one. Effective porosity is the percentage of the total mass of aquifer material occupied by interconnected pores. It is often estimated from site-specific data related to the composition of the aquifer material or the specific yield of the aquifer. **Note that a value other than zero should be entered for effective porosity even if hydraulic conductivity data are not available and calculation of groundwater velocity is not required.** Entering an effective porosity of zero will result in division by zero in the calculations and will render both Vector Inspector and Vector Plot unusable.

Date/Time and Hydraulic Heads

The date/time data are entered in "MM/DD/YY HR:MN" format (Figure 2). Hydraulic head data should be entered in the same length units as used for the well coordinates (e.g., feet, meters). Data may be entered manually or copied from electronic files using the "Paste Special…" and "Values" options. The number of rows of hydraulic head data for which hydraulic gradient and groundwater velocity will be calculated is only limited by the number of rows allowed in Excel®. Example rows consisting of input data followed by calculations and results are provided in the 3PE spreadsheet. The example input data in each row may be overwritten by the user. Following input of the hydraulic head data, the user should copy the calculation cells (i.e., 3PE spreadsheet columns E through AA) from one of the example rows and paste into the sheet (using the "Paste" option) as many times as needed based on the number of hydraulic head measurements to be solved as three-point problems for the given triangle of wells.

2.2 Results

The 3PE spreadsheet calculates and displays many useful parameters in addition to the magnitude and orientations of the hydraulic gradient and groundwater velocity vectors. These include data describing the triangle used in the three-point problem, vector components, and simple statistics describing the hydraulic head input data and the magnitudes of the calculated hydraulic gradient and groundwater velocity vectors. In this workbook, the symbol "i" is used to denote hydraulic gradient and the symbol "V" is used to denote groundwater velocity.

The results displayed in the spreadsheet are described below.

Statistics

The spreadsheet calculates the maximum, minimum, and average hydraulic head and the range of the hydraulic head data for each well. It also calculates the maximum, minimum, and average magnitudes of the hydraulic gradient and groundwater velocity vectors (Figure 3).

Vector Inspector

The hydraulic conductivity and velocity vectors for a selected row of hydraulic head data are displayed in the "Vector Inspector" chart as blue and red arrows, respectively (Figure 3). The starting and ending coordinates of each vector are provided in the cells labeled "PLOTTED HYDRAULIC GRADIENT ARROW COORDINATES" and "PLOTTED GROUNDWATER VELOCITY ARROW COORDINATES". For a quick view of the vectors for a particular Date/Time, the user enters the "Vector Inspector Row of Interest" (i.e., the spreadsheet row number where the Date/Time of interest is located). The range of rows available for viewing is shown below the Row of Interest cell. The "Vector Inspector" displays the triangle and the vectors. The scale factors for vectors in the "Vector Inspector" are functions of the magnitude of the vectors and the size of the problem grid. A recommended scale factor is provided. However, the user should adjust the scale factors to their satisfaction. Large problem grid sizes (e.g., large x and y coordinate values) may require large scale factors (e.g., > 10,000). Proper scaling will allow the workbook to display the vectors in both the "Vector Inspector" and as a separate plot under the workbook tab labeled "Vector Plot".

Number of Measurements

These cells calculate the total number of hydraulic head measurements (Figure 3) entered for each of the wells. The number of measurements should be the same for each well. If not, the user will receive the prompt: **WARNING: Please delete incomplete rows.**

Number of Vectors within Each Compass Quadrant

The numbers of hydraulic gradient vectors (i-vectors) and groundwater velocity vectors (*V*-vectors) indicating directions of groundwater movement in each of the four compass quadrants are displayed (Figure 4).

Triangle Information

Based on the coordinates of the triangle vertices, 3PE calculates the following information (Figure 4):

- The coordinates of the triangle centroid used as the starting point for the vectors.
- The area of the triangle.
- The length of the triangle sides.
- Angles of the triangle.

Triangle Plot Coordinates

The starting and ending coordinates of the line segments (the triangle sides) define the three wells used in the calculations (Figure 4). This information may be used in other plotting software for presentation purposes.

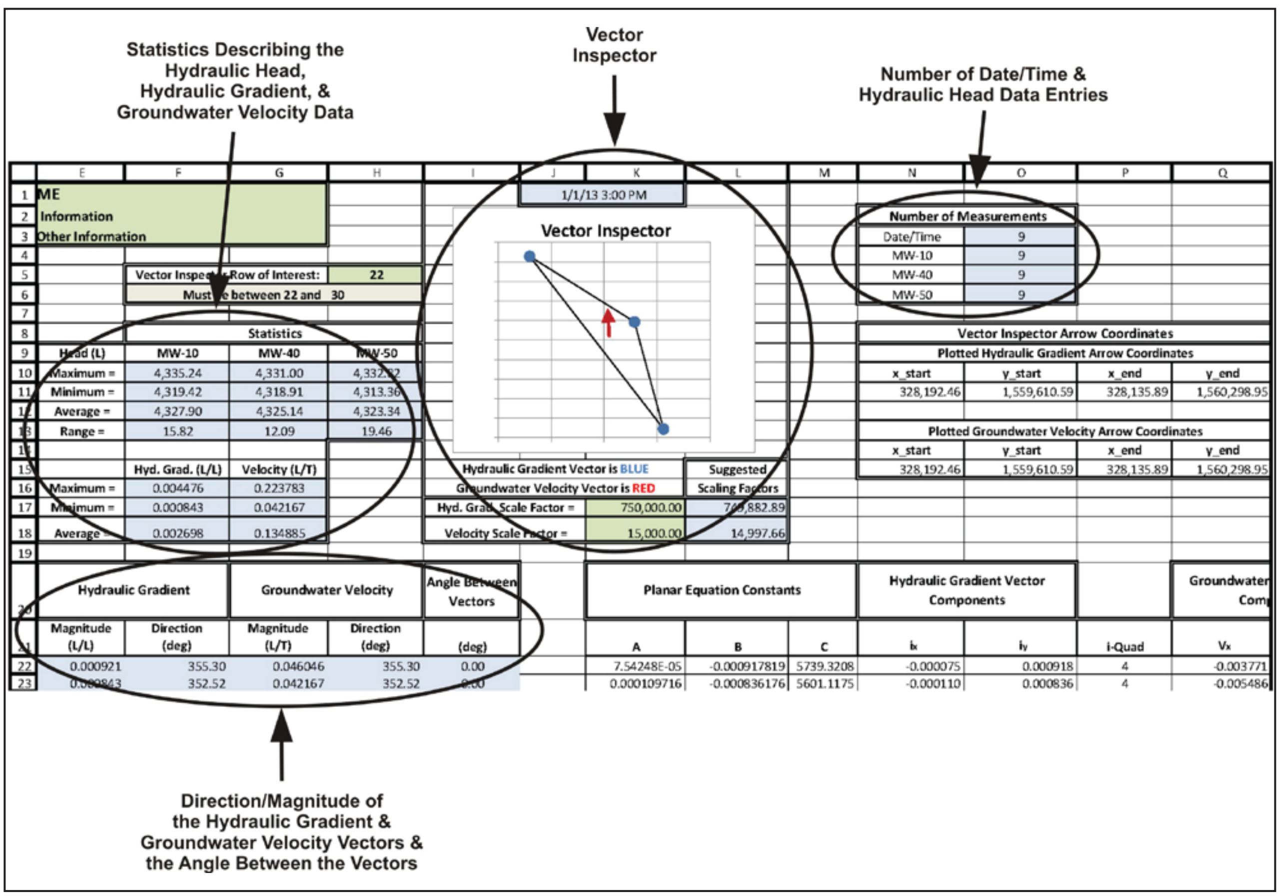

Figure 3. Screen capture from the 3PE spreadsheet depicting results of the hydraulic gradient and groundwater velocity vector calculations and the Vector Inspector feature. Cells for user input are shaded green. Blue cells are the most significant results of the calculations.

Hydraulic Conductivity Components

The principal horizontal hydraulic conductivity components (i.e., the maximum horizontal hydraulic conductivity (K_{max}) and the minimum horizontal hydraulic conductivity (K_{min})) and the orientation of K_{max} measured in degrees clockwise from North are used to calculate the components of the hydraulic conductivity tensor (i.e., K_{xx},K_{yy} for anisotropic conditions and K_{xy}=K_{yx} for isotropic conditions) (Figure 4). The rotation angle (θ) is calculated as the angle between the x-axis and the K_{max} axis (See Appendix A.).

Individual Results for Each Three-Point Problem

The 3PE spreadsheet calculates and displays the following results for each row of hydraulic head input:

- Hydraulic Gradient Magnitude (Column E) - Magnitude of hydraulic gradient (L/L).
- Hydraulic Gradient Direction (Column F) - Orientation of hydraulic gradient in degrees clockwise from North (positive y-axis).
- Groundwater Velocity Magnitude (Column G) - Magnitude of groundwater velocity vector (L/T).
- Groundwater Velocity Direction (Column H) - Orientation of groundwater velocity vector in degrees clockwise from North (positive y-axis).
- Angle Between Vectors (Column I) - Difference between orientations of hydraulic gradient (i) and groundwater velocity (V) vectors in degrees.
- A, B, C (Columns K-M) - Constants of the equation defining a plane solved to obtain hydraulic gradient.
- i_x (Column N) - Hydraulic gradient component in x direction (L/L).
- i_y (Column O) - Hydraulic gradient component in y direction (L/L).
- i-Quadrant (Column P) - One of four regions of the two-dimensional Cartesian system, bounded by two half-axes. In clockwise order, the quadrants are 1 (azimuth from 0 to 90 deg); 2 (azimuth from 90 to 180 deg); 3 (azimuth from 180 to 270 deg); and 4 (azimuth from 270 to 360 deg).
- V_x (Column Q) - Groundwater velocity component in the x direction (L/T).

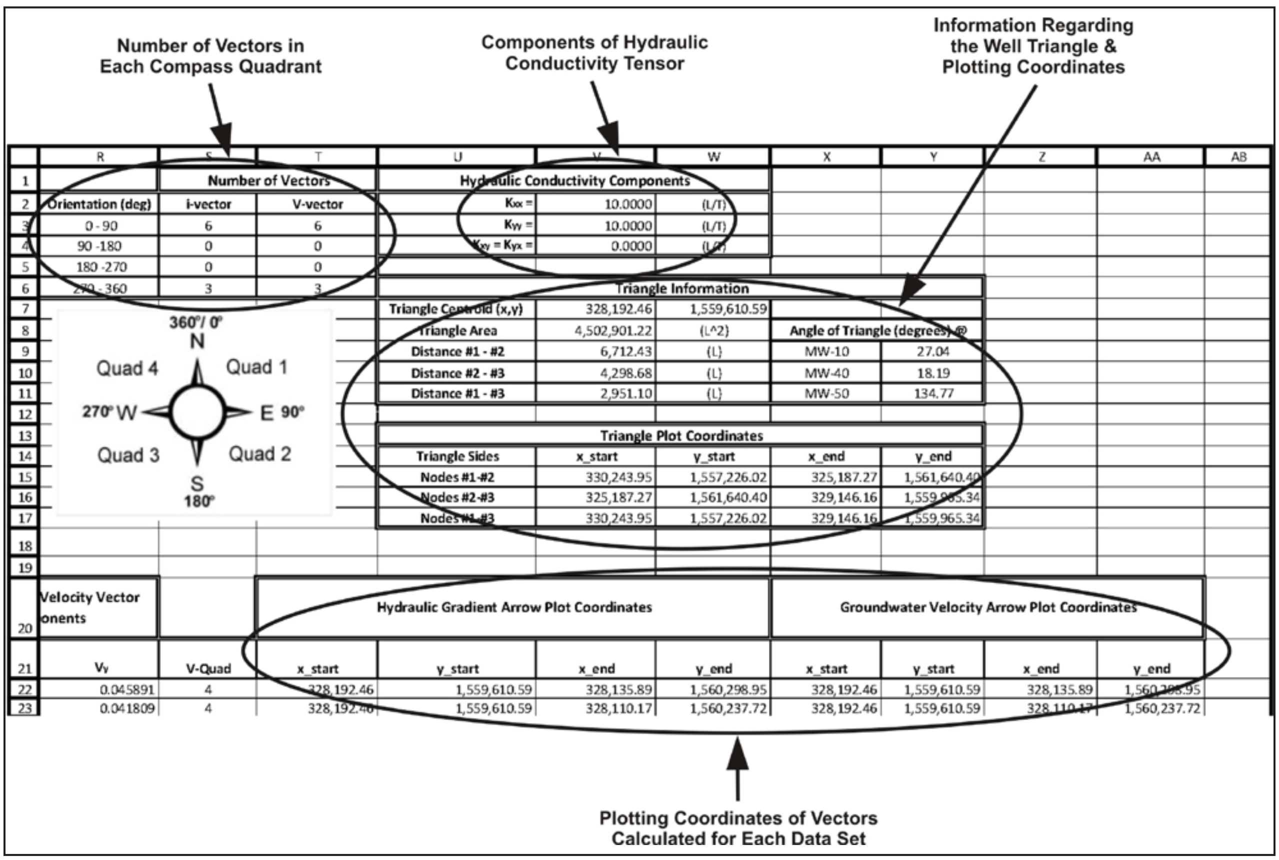

Figure 4. Screen capture from the 3PE spreadsheet depicting results describing the orientations of the vectors, the triangle of wells used to calculate hydraulic gradients, the components of the horizontal hydraulic conductivity tensor, and the vector plotting coordinates calculated for each data set.

- V_y (Column R) - Groundwater velocity component in the y direction (L/T).
- V-Quadrant (Column S) - One of four regions of the two-dimensional Cartesian system, bounded by two half-axes. In clockwise order, the quadrants are 1 (azimuth from 0 to 90 deg); 2 (azimuth from 90 to 180 deg); 3 (azimuth from 180 to 270 deg); and 4 (azimuth from 270 to 360 deg).
- Hydraulic Gradient Arrow Plot Coordinates x_start (Column T) - Starting x coordinate for hydraulic gradient arrow plot.
- Hydraulic Gradient Arrow Plot Coordinates y_start (Column U) - Starting y coordinate for hydraulic gradient arrow plot.
- Hydraulic Gradient Arrow Plot Coordinates x_end (Column V) - Ending x coordinate for hydraulic gradient arrow plot.
- Hydraulic Gradient Arrow Plot Coordinates y_end (Column W) - Ending y coordinate for hydraulic gradient arrow plot.
- Groundwater Velocity Arrow Plot Coordinates x_start (Column X) - Starting x coordinate for groundwater velocity arrow plot.
- Groundwater Velocity Arrow Plot Coordinates y_start (Column Y) - Starting y coordinate for groundwater velocity arrow plot.
- Groundwater Velocity Arrow Plot Coordinates x_end (Column Z) - Ending x coordinate for groundwater velocity arrow plot.
- Groundwater Velocity Arrow Plot Coordinates y_end (Column AA) - Ending y coordinate for groundwater velocity arrow plot.

2.3 Hydraulic Head and Gradient Plot

On the Hydraulic Head and Gradient Plot tab, the workbook contains a plot of the direction of the hydraulic gradient vector and the hydraulic head data for each well as a function of the Date/Time input in each data row in the 3PE spreadsheet. The plot may be edited, as needed, and exported in portable document format (PDF) for importation into slide show presentation or other software, if desired. The plot provides a rapid means of identifying significant changes in the direction of the hydraulic gradient through time.

2.4 Hydraulic Gradient Plot

On the Hydraulic Gradient Plot tab, the workbook contains a plot of the direction and magnitude of the hydraulic gradient vector as a function of the Date/Time input in each data row in the 3PE spreadsheet. The plot may also be edited and exported for presentation. The plot provides another tool for the rapid visualization of any temporal trends in the hydraulic gradient that may be seasonal or related to other hydrologic changes at the site.

2.5 Groundwater Velocity Plot

In similar fashion, the direction and magnitude of the groundwater velocity vector is also plotted on a workbook tab.

2.6 Vector Plot

A plot of the hydraulic gradient and groundwater velocity vectors displayed in the "Vector Inspector" section of the 3PE spreadsheet is included on the Vector Plot tab to allow the user to more easily manipulate the plotting functions and export the plot, if desired. "Vector Plot" can be enhanced by adjusting the axes, re-positioning the well labels, changing the chart title, etc. Once the chart options are selected, a number of charts can be generated by selecting a different "Vector Inspector Row of Interest" in Row 5, Column H of the 3PE worksheet (i.e., Date/Time).

2.7 User Instructions

A simplified version of the instructions for use of 3PE is provided on the first workbook tab.

3.0
Application of the Three-Point Method of Analysis

The hydraulic gradient is usually estimated using synchronous groundwater elevation measurements from wells and piezometers (Figure 5). Estimates of the direction and magnitude of the hydraulic gradient in a given part of the aquifer may then be used with estimates of the hydraulic conductivity and the effective porosity to characterize the direction and rate of groundwater flow (i.e., groundwater seepage velocity) using a form of Darcy's Law. More extensive discussions of the fundamental concepts of groundwater flow are available from a variety of textbooks (e.g., Freeze and Cherry, 1979; Domenico and Schwartz, 1990; Fetter, 1988) and other publications (e.g., USEPA, 1990).

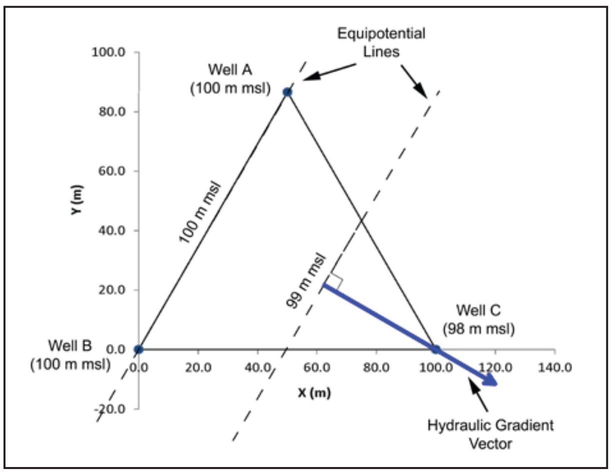

Figure 5. Calculation of hydraulic gradient vector using hydraulic head data from three locations (Wells A, B, and C). The hydraulic gradient vector, calculated graphically using the methods of Heath (1983) in this case, is perpendicular to contours of equal hydraulic head (equipotential lines) and is the direction of groundwater flow in an isotropic setting.

One of the basic tools for evaluating hydraulic gradients on a site-wide scale is the potentiometric surface map (Figure 6). A potentiometric surface represents the level to which water would rise in wells (Bates and Jackson, 1987). It is produced either manually, or using specialized software to plot and contour groundwater elevation measurements that are representative of a single hydrostratigraphic unit and obtained within a limited timeframe. Following the convention used in much of the hydrogeological literature (e.g., Bates and Jackson, 1987; Fetter, 1988), the water table is considered to be a particular potentiometric surface for purposes of the discussions in this document. Assuming a horizontally isotropic aquifer without a significant component of vertical groundwater movement, groundwater flows in directions perpendicular to potentiometric surface contours (i.e., equipotential lines) in the direction of decreasing hydraulic head (i.e., direction of the hydraulic gradient vector).

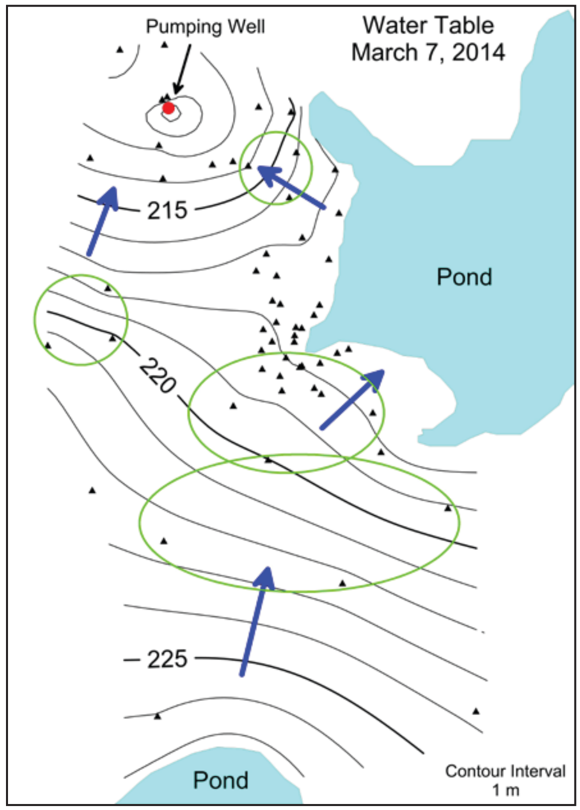

Figure 6. Example potentiometric surface map interpreted from hydraulic head data measured in multiple wells (black triangles). In general, groundwater flows from areas where it enters the aquifer to areas where it leaves the aquifer. Blue arrows depict groundwater flow directions indicated by hydraulic gradients. Green circles and ellipses highlight examples of areas where the water table is sufficiently planar and wells exist for analysis of hydraulic gradients using three-point methods.

Examination of the potentiometric surface often reveals basic characteristics of the groundwater flow field that control groundwater flow and the transport of any dissolved constituents (Figure 6). In general, groundwater flows from recharge zones where water enters an aquifer, such as locations with significant infiltration from precipitation or surface water, to discharge zones where water leaves the aquifer, such as pumping wells, seeps, springs, and surface water bodies. Recharge zones may be reflected on potentiometric surface maps as areas with higher groundwater elevations and increased hydraulic gradient (i.e., smaller spacing between equipotential lines). In similar fashion, groundwater discharge zones may be indicated by significant depression of the potentiometric surface associated with groundwater discharge (e.g., near extraction wells or discharge to surface water). Examination of potentiometric surfaces obtained at different points in time, such as wet and dry seasons, or when local water supply wells are pumping at different rates, can illustrate temporal fluctuations in hydraulic gradients due to changes in hydrologic conditions.

Where sufficient hydraulic head data are available to define the potentiometric surface, portions of the aquifer where groundwater flow is planar will be reflected as areas where the equipotential lines are straight, rather than curved, and have a relatively uniform spacing (Figure 6). In areas where groundwater flow is planar, the hydraulic gradient can be estimated using groundwater elevations from a minimum of three wells that form a triangle. Calculation of the hydraulic gradient using hydraulic heads measured in three wells has been referenced in the literature by terms such as the "three-point problem" or "three-point method". The problem may be solved graphically (Figure 5) (Heath, 1983) or using more advanced mathematical techniques (e.g., Pinder et. al, 1981; Cole and Silliman, 1996; Silliman and Frost, 1998; Devlin and McElwee, 2007). Although calculation of hydraulic gradient using more than three wells can decrease uncertainty in some situations, such as in areas where the hydraulic gradient is low, the benefits of including more than three monitoring points are often small in comparison with other methods for reducing uncertainty (e.g., increasing well spacing). In practice, the three-point method is often a useful and cost effective tool, particularly for initial investigations (Devlin and McElwee, 2007).

3.1 Hydrogeologic Considerations

As with every evaluation technique, there are assumptions that should be met before the three-point method is applied. Some of these assumptions are specific to the three-point method while others are also applicable to tools commonly used for estimation of hydraulic gradients (e.g., potentiometric surface maps). Key assumptions specific to the three-point method and use of the 3PE spreadsheet are:

- **The water table or other potentiometric surface within the triangle of wells can be represented as a plane (i.e., the curvature of the potentiometric surface is relatively small).**

  Conditions under which the potentiometric surface is unlikely to be planar include situations where there are point sources for water movement into or out of the aquifer located within the triangle. For example, surface water bodies often serve as either recharge zones or discharge zones for the aquifer. Similarly, active pumping wells screened within or hydraulically communicating with the aquifer of interest are obvious sources of discharge. The effects of improperly applying three-point analysis or other simple methods to estimate hydraulic gradients in such areas can be dramatic (Figure 7). In this example, an active pumping well was included within the triangle of monitoring wells used to estimate groundwater flow directions.

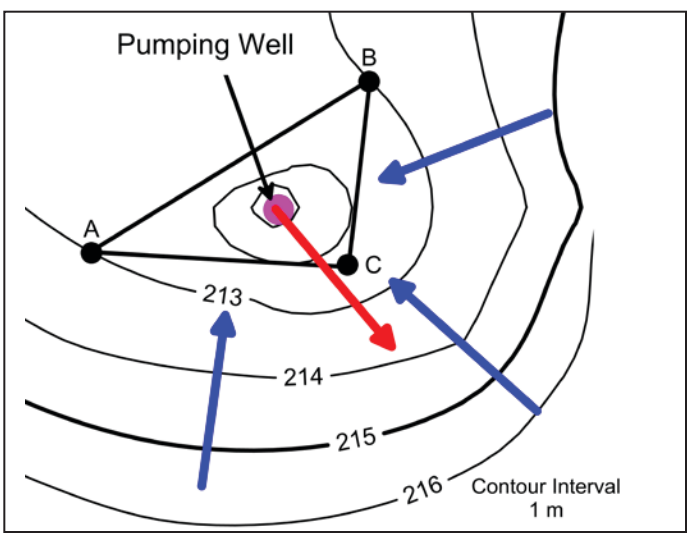

Figure 7. Misapplication of a three-point analysis of groundwater flow direction surrounding an actively pumping well. The contours of the potentiometric surface reflect the non-planar cone of depression associated with the pumping well. Monitoring wells used to calculate the hydraulic gradient by the three-point method are labeled A, B, and C. The red arrow depicts the groundwater flow direction calculated using three-point analysis and the blue arrows depict the actual groundwater flow direction. In this case, improper application of the three-point analysis results in a calculated flow direction that is opposite to the actual direction.

The curvature of the potentiometric surface is significant in this area indicating it is not planar. The red arrow depicts the groundwater flow direction calculated using three-point analysis and the blue arrows depict the actual groundwater flow direction. In this case, improper application of the three-point analysis results in a calculated flow direction that is opposite to the actual direction. Other less obvious situations, such as areas with enhanced infiltration or leaking water conveyance systems, can result in significant curvature of the potentiometric surface. It is recommended that such features be considered during the choice of wells used for estimation of hydraulic gradients by the three-point method. Where sufficient data are available, potentiometric surface maps should be used to aid in determining areas of a site where planar groundwater flow conditions likely exist.

- **The aquifer can be treated as homogeneous within the triangular area used for the three-point analysis.**

  Geologic heterogeneity that results in significant changes in hydraulic conductivity can result in significant changes in hydraulic gradients. Such conditions also make specification of representative values for hydraulic conductivity and effective porosity subject to increased uncertainty. If this occurs within the area of the three-point problem, it is likely that the resulting hydraulic gradient and groundwater velocity vectors will not accurately represent actual site conditions. Evaluations of geologic logs and estimates of hydraulic conductivity obtained from the wells defining the three-point problem as well as consideration of the geologic setting can be used to evaluate the degree of heterogeneity that may exist. Evaluation of potentiometric surfaces may also reveal areas with changes in hydraulic gradients that may be related to heterogeneity. In general, effects due to heterogeneity may decrease as the size of the three-point triangle increases relative to the scale of the heterogeneity (Cole and Silliman, 1996; McKenna and Wahi, 2006). However, increasing the size of the triangle must be balanced by the requirement that groundwater flow within the triangle be planar in order to apply three-point analysis methods.

Key assumptions related to hydrogeology and well construction that are applicable to most simple tools used to estimate horizontal hydraulic gradients, including the three-point method, are:

- **The wells are screened solely in the same aquifer or hydrostratigraphic unit.**

  The distribution of hydraulic head and, therefore, groundwater flow directions and rates may be very different within different aquifers. Consider the situation (Figure 8) where one well (Well A) is screened at the water table, another well (Well B) is screened within a deeper, confined aquifer, and the third well (Well C) is screened across the confining unit separating the two aquifers. For this example, the hydraulic head in the upper aquifer is higher than the head in the lower aquifer. The water level in Well A is higher than the water level in Well B and the water level measured in the well screened across the confining unit (Well C) is influenced by both aquifers but is representative of neither aquifer.

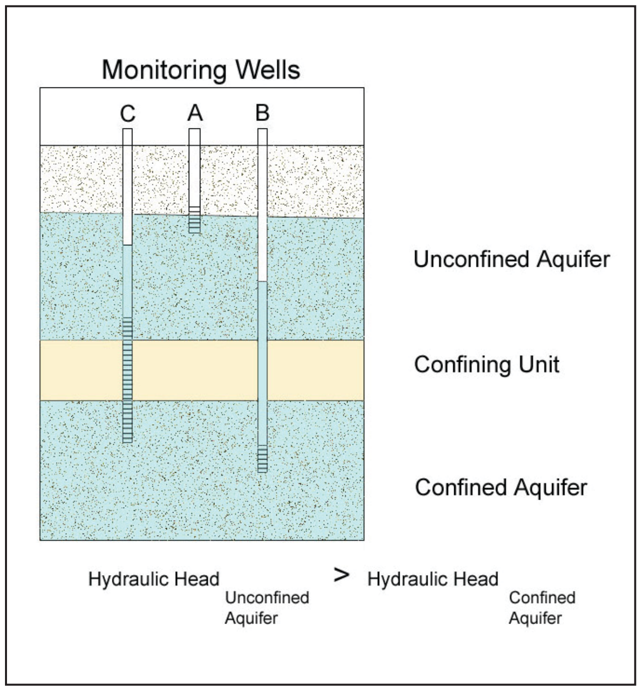

Figure 8. Groundwater elevations in three adjacent wells (A, B, and C) screened at different depths. In this case, Well A is screened across the water table in an unconfined aquifer, Well B is screened in a deeper confined aquifer, and Well C is screened in both aquifers. Differences in hydraulic head in the two aquifers result in significant differences in the groundwater elevations measured in the wells.

Three-point analysis methods, such as implemented in the 3PE spreadsheet, would not be appropriate for use with this well network. Use of data from wells screened within different aquifers or across confining units can result in estimated groundwater flow directions and rates that are completely unrepresentative of actual conditions. As a general practice, the wells should be of similar screen length and position within the aquifer. In addition, the use of wells with long screens should be avoided to minimize the potential for screening zones with different hydraulic heads in order to provide data most representative of aquifer conditions. In situations where hydraulic gradients are to be calculated for a confined aquifer, lack of effective borehole seals can result in vertical water flow outside the well casing and water-level measurements that are not representative of hydraulic head in the screened portion of the aquifer. Information concerning local geology and aquifer structure, as well as construction logs for the wells to be used to estimate hydraulic gradients, should be evaluated to insure all wells are properly constructed and screened solely within the aquifer or hydrostratigraphic unit of interest.

- **Groundwater elevation measurements are synchronous.**

  At many sites, potentiometric surfaces and hydraulic gradients change with time due to changes in hydrologic conditions. These changes can be due to such things as aquifer recharge from precipitation, changes in pumping rates of nearby water supply wells, or changes in nearby surface water elevations. The frequency of such fluctuations should be considered during planning of field activities to insure water levels are measured synchronously. Groundwater and surface water elevations used to estimate hydraulic gradients should be measured within a timeframe such that they are all representative of the same point in time (i.e., no significant changes occur during the measurement period). For example, if several days were required to conduct manual measurements of groundwater elevations at a site and rainfall resulted in significant increases in water levels during the measurement period, hydraulic gradients estimated using this data set may not be representative of site conditions. In this situation, it may be necessary to identify smaller regions within the monitoring network to allow manual water level measurements within a time period suitable for application of three-point problem analysis. As a general practice, groundwater elevation measurements should be obtained from all wells used to create potentiometric surfaces or estimate hydraulic gradients in as short a timeframe as practical. Data from pressure transducers can be used to characterize the frequency of significant temporal changes and allow more informed planning of future field activities.

  Other important sources of transient behavior or time lag effects that should be considered are discussed by Post and Asmuth (2013). These include the well-specific response time following a change in groundwater pressure due to well volume, screen length, and permeability of materials adjacent to the well screen. This may be particularly important in low permeability settings and where fluctuations in water elevations are rapid, such as near a beach where wave action results in periodic changes in water elevations. Inadequate well development may also result in similar time lag effects.

  The effects of time lags related to barometric pressure changes should also be considered. In an unconfined aquifer, significant delays in the transmission of atmospheric pressure changes to the water table may be observed if the water table is deep or the vadose zone has a low permeability to air (Post and Asmuth, 2013). In such situations, correction of water-level measurements for barometric pressure effects should be evaluated and implemented if needed to reduce uncertainty.

- **Groundwater flow is predominantly horizontal.**

  Hydraulic gradients estimated using the three-point method or other simple tools generally represent the horizontal component of the hydraulic gradient. In a setting where hydraulic head within the aquifer of interest changes with depth, there will also be a vertical component to the hydraulic gradient and, potentially, a significant vertical component of groundwater flow. In settings where a vertical hydraulic gradient exists and vertical hydraulic conductivity is not significantly lower (e.g., two orders of magnitude) than horizontal hydraulic conductivity, a significant component of vertical flow may exist and should be considered prior to application of simple three-point analysis methods. More complex analyses would be needed to fully characterize the three-dimensional aspects of groundwater flow in situations where the vertical flow is not negligible.

3.2 Data Collection Tools and Techniques

Various tools are commonly used for the accurate measurement of water level in wells. In general practice, measurements are made with respect to a surveyed elevation which, for monitoring wells or piezometers, is usually a reference point marked on the top of the well casing. This allows measurements of the depth to groundwater to be expressed as hydraulic head relative to a common datum, which is required for calculation of hydraulic gradients and most other hydrogeologic analyses.

Submersible pressure transducers that automatically compensate for temperature combined with data loggers for storing the measurements have become more portable, reliable, and affordable. In some situations, the use of pressure transducers allows a cost-effective way of collecting a large amount of hydraulic head data with a pre-programmed time interval, ensuring that all measurements are made synchronously. This makes pressure transducers well suited for use in investigations of temporal changes in hydraulic gradients that occur on a time scale that is impracticable to fully characterize using manual measurements of groundwater elevations.

Pressure transducers commonly used in groundwater monitoring are either vented to the atmosphere by a vented cable (gauged transducer), which means the readings are automatically compensated for barometric pressure, or unvented (absolute transducer), where the pressure transducer measures the total pressure (i.e., atmospheric pressure plus the pressure of the water column above the transducer). When using an unvented transducer, site-specific barometric pressure must also be measured and subtracted from the total (absolute) pressure reading to obtain the water pressure. If groundwater elevation is independently measured coincident with pressure transducer measurements, the transducer data can then be converted to groundwater elevations using methods described in the operations manuals that accompany most instruments.

Pressure transducers are designed to be used under specified pressure ranges. The accuracy and precision of the transducer are often a function of the transducer range. As the transducer range decreases, the uncertainty in the measurement due to the accuracy and precision of the transducer often decreases. For example, the reported accuracy and precision of current absolute pressure instruments with a pressure range below approximately five meters of water is often +/- a few millimeters of water. The accuracy and precision of similar instruments with a much higher pressure range (e.g., 100 meters of water) may be +/- a few centimeters of water. Product specifications provided by the manufacturer should be consulted to determine the accuracy and precision of particular pressure transducers. Previous site-specific monitoring data should be consulted to estimate the range of water levels that may be observed. This information will allow the optimum pressure range for the transducer to be specified.

The user should also consider transducer performance in the areas of temperature compensation and pressure sensor drift (Figure 9). Historically, transducer drift (i.e., deterioration in sensor accuracy over time) and its potential effects on hydrogeologic studies have been noted (e.g., Rosenberry, 1990). Recently, Sorensen and Butcher (2011) describe the results of independent tests of pressure transducers used for water-level monitoring. Pressure sensor drift was judged to be significant for some of the tested transducers over the course of a 99-day field test. Drift was found to be linear in some instances and nonlinear in others. In addition, their results indicate that the degree to which the instruments accurately compensated for changes in temperature varied. Unfortunately, product specifications often do not describe transducer performance with respect to sensor drift or temperature compensation. Therefore, assessment of these instrument characteristics by the user may be needed in some situations, such as the long-term monitoring of water levels where a high degree of accuracy is required. In addition, periodic manual measurements of water levels will generally be needed during field deployment of either vented or unvented transducers to confirm that transducer accuracy meets project requirements and, potentially, correct the data for sensor drift.

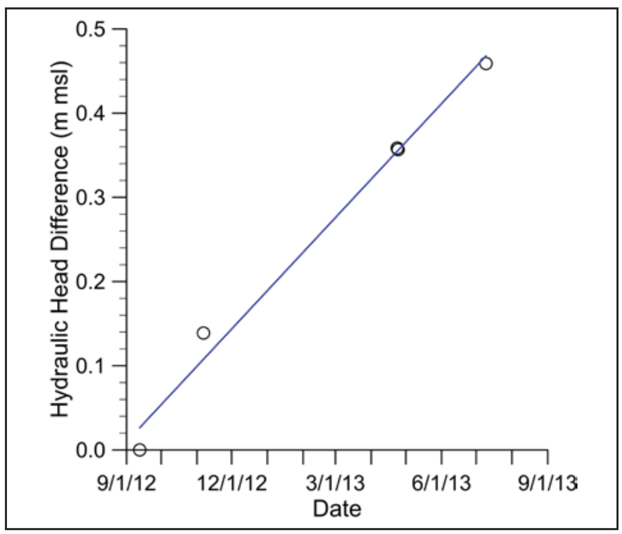

Figure 9. Example of pressure sensor drift unrelated to temperature compensation issues during extended field deployment of a pressure transducer. The plot depicts the difference between hydraulic head measured using a pressure transducer installed in a monitoring well and hydraulic head synchronously measured using an electric water-level indicator in the same well (open circles). In this case, instrument drift steadily increased throughout the deployment. The blue line is a simple linear regression of the data.

3.3 Assessment of Measurement Uncertainty in Hydraulic Gradient Analysis

Measurement uncertainty can have a significant impact on the accuracy of hydraulic gradients calculated using three-point methods (Silliman and Mantz, 2000). Awareness of the sources of measurement uncertainty or error and the magnitude of the uncertainty is often essential to the evaluation of hydraulic gradients, design of the monitoring network, and decisions concerning instrumentation. Uncertainty in hydraulic head measurements is introduced to varying degrees by each part of the measurement process, including uncertainty due to the accuracy/precision of the measurement device, the measurement technique, and even the specification of reference elevations.

3.3.1 Measurement Devices

Devices commonly used for the measurement of water levels in wells include the steel tape, electric water-level indicator, air line, float-activated recorder, and submersible pressure transducers. Each tool has advantages and disadvantages with respect to the others. In addition, there is measurement uncertainty inherent in the use of each device. In many instances, the measurement uncertainty associated with the device can be readily quantified. For many devices, the accuracy is often specified by the manufacturer. For example, most current electric water-level indicators are marked to 0.01 ft or 1 mm, calibrated, and traceable to national standards. Periodic comparison of a water-level indicator with a reference tape can be used to ensure calibration is maintained and documented.

Similarly, the accuracy and precision of pressure transducers used to measure hydraulic head are generally stated by the manufacturer and can be significantly greater than the measurement uncertainty associated with manual measurements of water level. In addition, sensor drift and inadequate temperature compensation can reduce accuracy under some field conditions, such as long-term deployments and conditions where temperatures vary significantly. When using unvented (absolute) transducers, there will be measurement uncertainty associated with both the submerged transducer and the barometric pressure transducer or other instrument used to measure barometric pressure that will be cumulative and should be considered.

The choice of appropriate devices for measuring hydraulic head at a given site should consider several factors, including:

- Study objectives (i.e., what question is to be answered).
- Measurement accuracy required to meet the objectives.
- Measurement accuracy of the device and proposed measurement technique.
- Advantages, disadvantages, and limitations of the device.

3.3.2 Measurement Procedures

The thoughtful development of standard operating procedures for obtaining the data used to calculate hydraulic head and estimate hydraulic gradients is a key element in controlling measurement uncertainty. The procedures should identify and quantify the various sources of uncertainty associated with the particular data collection instruments and provide procedures to control the uncertainty. For example, proper techniques for obtaining manual water-level measurements in a monitoring well using an electric water-level indicator generally include:

- Measuring the depth to water with respect to a surveyed reference mark with the required accuracy (e.g., nearest 1 mm or nearest 0.01 ft).
- Recording of the values in a notebook that meets quality assurance standards.
- Obtaining duplicate measurements in all or a specified percentage of the wells.
- Insuring that the water level in the well is stable following insertion of the monitoring probe, particularly in wells with a small internal diameter (e.g., less than 5 cm), in situations where the well may be screened in aquifer materials with a low to moderate hydraulic conductivity, and in wells that may not be adequately vented to the atmosphere.
- Procedures for testing the instrument battery and for regular battery replacement.
- Site-specific procedures for decontaminating the probe between wells.
- At sites where the water density is variable, procedures for head corrections to account for density effects may be needed (Post et al., 2007).
- At sites where light nonaqueous phase liquid (LNAPL) may be present, procedures to measure LNAPL thickness and correct hydraulic head measurements for LNAPL presence may be needed (Newell et al., 1995).

Procedures for obtaining data using submersible pressure transducers should include:

- Specification of transducer range. Consideration should be given to the required measurement accuracy as well as the likely range of water levels that will be observed during the measurement period. Ideally, the specified pressure range of the instrument should be large enough to allow measurements of the highest and lowest projected water levels but still provide data with the required accuracy. When available, previous data concerning the temporal patterns of water-level fluctuations should be used to determine both the appropriate transducer and the elevation within the water column to suspend the transducer to insure data are obtained over the full range of water levels. Consideration should also be given to placement at a depth such that the transducer remains both submerged and within its pressure range throughout deployment.

- Procedures for suspension of the transducer in the well to insure that the transducer remains at a constant elevation throughout the investigation. Consideration should be given to the point of attachment on the well and use of a suspension line that will not stretch or corrode (e.g., stainless steel cable).
- Procedures for measurement of barometric pressures synchronous with the submerged transducer readings if absolute pressure (unvented) transducers are used. Barometric pressure can be measured using a specialized barometric pressure transducer installed on-site. Alternatively, barometric pressure data can be obtained from other measurement devices such as an on-site or nearby barometer. Consideration should be given to installation of a backup measurement device for monitoring barometric pressure, since it is a critical parameter for correcting data from absolute pressure transducers.
- Procedures for programming data acquisition, including synchronization of instrument clocks, and for downloading and archiving the data.
- In most situations, the well should be vented to allow equilibration of the internal casing with atmospheric pressure.
- For vented transducers, inspection of the transducer vent to ensure it is open to the atmosphere and routine replacement of the desiccant capsule, if so equipped.
- Corrections for water density due to temperature or salinity where such corrections are warranted.
- Periodic manual measurements of hydraulic head. These data are needed to convert the pressure measurements to groundwater elevations and to evaluate instrument drift.
- Routine checks of instrument calibration with recalibration, as needed.

A succinct discussion of water-level measurements and recommended procedures for use of various devices (e.g., steel tape, electric indicator, air line, float-activated recorder, submersible pressure transducers) for measuring water levels in wells is available in Cunningham and Schalk (2011).

3.3.3 Reference Elevations

The accuracy of the surveyed elevation of the reference point used to calculate groundwater elevation should always be specified (e.g., +/- 0.003 m or +/- 0.01 ft). However, significant errors in the specified reference elevations can occur, particularly at sites with a complex characterization history, due to a variety of conditions, including:

- Wells installed in phases were surveyed using a different reference datum.
- Transcription errors occurred and propagated.
- Well is misidentified in the field.
- Top of well casing is uneven and has no marked survey reference point.
- Reference elevation has changed due to conditions such as frost heaving.
- Survey used poor technique or lack of accuracy control.

Errors in the specified reference elevations are often more difficult to evaluate than uncertainties associated with the measurement device or measurement technique. Such errors are sometimes evidenced by obviously abnormal or odd groundwater elevation measurements that are not readily interpretable. In situations where a high degree of measurement accuracy is required, careful examination of survey history and resurvey of the reference elevations at key wells may be warranted.

3.3.4 Monitoring Network Design

As with most methodologies, measurement uncertainty should be considered in the design of the monitoring network used for three-point analysis of hydraulic gradients. The overall uncertainty in the measurement is the accumulation of uncertainties related to the various investigation-specific sources. Data quality objectives providing a quantitative, as well as qualitative, description of the data quality required to support the proposed analyses and decisions should be developed for each investigation (Post and Asmuth, 2013). Detailed discussion regarding the development of data quality objectives is provided in USEPA (2006). Following specification of the required data quality, measurement methods and a monitoring design capable of obtaining the desired accuracy can be chosen and measurement uncertainty quantified. Quantifying measurement uncertainty is particularly important in a setting where the hydraulic gradient is low or the distance between wells is small. In such situations, the differences in hydraulic head among the wells may be very small. Measurement uncertainty may be too large to allow estimation of hydraulic gradients with the required degree of accuracy. In which case, use of other measurement methods or a larger well spacing may be needed to reduce uncertainty.

With respect to design of monitoring networks for hydraulic gradient analysis using the three-point method, the following guidelines are provided by Devlin and McElwee (2007) and McKenna and Wahi (2006):

- The measured groundwater elevation differences between wells should be much greater than the expected measurement uncertainty to produce reliable estimates of hydraulic gradients. This implies that it may be necessary to increase the spacing between measurement points in some situations, particularly in settings where the hydraulic gradient is low. The degree to which the measured water-level difference should exceed the measurement uncertainty will be site specific and depend on the data quality requirements of the investigation. For example, studies reported by Devlin and McElwee (2007) at the Geohydrologic Experimental and Monitoring Site, an alluvial aquifer study site with a low hydraulic gradient (approximately 0.0005), resulted in a site-specific guideline that the monitored area should be large enough that the head drop across the area would be at least three times the expected measurement uncertainty. This guideline provided data of sufficient quality to meet the particular study objectives. In similar fashion, McKenna and Wahi (2006) recommend the use of the relative head measurement error (RHME) in designing the monitoring network for use with three-point solution methods. The RHME is defined as measurement error normalized by the head drop across the three-point estimator. The RHME can be used as an objective means for evaluating the minimum size of the three-point estimation triangle.
- Distortion of the triangle formed by the three monitoring points can result in increased uncertainty. Ideally, the ratio of any two of the sides of the triangle would be close to 1. The dimension of the triangle perpendicular to the direction of the hydraulic gradient can be increased in order to reduce the uncertainty in the estimate of the hydraulic gradient direction (Devlin and McElwee, 2007). McKenna and Wahi (2006) found that triangles with base-to-height ratios between 0.5 and 5.0 resulted in the most accurate gradient estimates.

4.0
Example Applications

Brief examples of the application of three-point solution methods and the 3PE spreadsheet are discussed below.

4.1 Application #1: Characterization of Nearby Extraction Well Effects

At some sites, groundwater flow and contaminant transport or the effectiveness of a remediation system may be affected by pumping of off-site wells. The following example illustrates the use of the 3PE spreadsheet in characterizing such effects.

4.1.1 Background and Setting

The site is a closed industrial facility situated in a valley, overlying an unconsolidated aquifer composed of alluvial fan and flood-plain deposits. Estimates of hydraulic conductivity within the alluvial aquifer range from less than 0.3 m/d to approximately 50 m/d. In general, groundwater flows in a north to northwest direction from the site. Groundwater contamination associated with the facility has migrated off site, extending approximately 1.5 km north of the facility boundary. Implementation of hydraulic controls to contain the plume are currently being considered. During the characterization process, it was discovered that a high-capacity irrigation well located approximately 3 km northeast of the site is active during the agricultural growing season.

4.1.2 Characterization Objective

The investigation was designed to answer the following question:

Does agricultural pumping affect groundwater flow directions within the contaminant plume?

4.1.3 Network Design and Data Acquisition

For the initial investigation of the possible effects of agricultural pumping on groundwater flow within the plume, three existing wells screened at similar elevations within the aquifer were instrumented with pressure transducers to synchronously measure hydraulic heads six times daily for approximately nine months (January through September). The wells were located near the downgradient margin of the plume. Groundwater elevations were also measured monthly using an electric water-level indicator to document the accuracy of the transducer measurements and allow evaluation of instrument drift.

4.1.4 Results and Conclusions

The magnitude and direction of the hydraulic gradient were calculated using 3PE for each of the approximately 1,600 data sets obtained during the monitoring period. The results indicate that the irrigation well is, potentially, a major influence on groundwater flow near the downgradient margin of the contaminant plume (Figure 10). Prior to the onset of the irrigation season, the average direction of the hydraulic gradient and, therefore, groundwater flow was approximately north. During this period, the average magnitude of the hydraulic gradient was approximately 0.0009 m/m. During the irrigation season, which began in mid-March, the hydraulic gradient was oriented in a northeast direction toward the irrigation well with an average magnitude of 0.0035 m/m, which is approximately four times larger than the magnitude of the hydraulic gradient under non-pumping conditions.

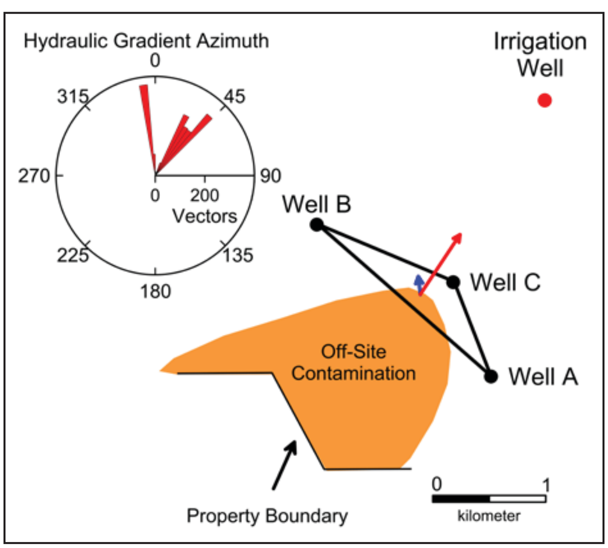

Figure 10. Influence of irrigation pumping on groundwater flow at downgradient limits of a contaminant plume. Hydraulic gradients were calculated using the three-point method implemented in 3PE with hydraulic head data obtained from wells A, B, and C. The distribution of the vector directions during the monitoring period is presented using a rose diagram. The average hydraulic gradient vector during the irrigation season (red vector) is significantly different from the average vector while pumping is not occurring (blue vector). The directions of the arrows indicate the groundwater flow directions. The difference in the magnitudes of the hydraulic gradients is indicated by the relative lengths of the vector arrows.

The direction and magnitude of the hydraulic gradient change abruptly with the onset of pumping (Figure 11). The magnitude of the gradient also appears to increase as the agricultural season progresses. Based on these initial results, it was determined that agricultural pumping may be a major influence on groundwater flow and would require both additional characterization and consideration during remedial design.

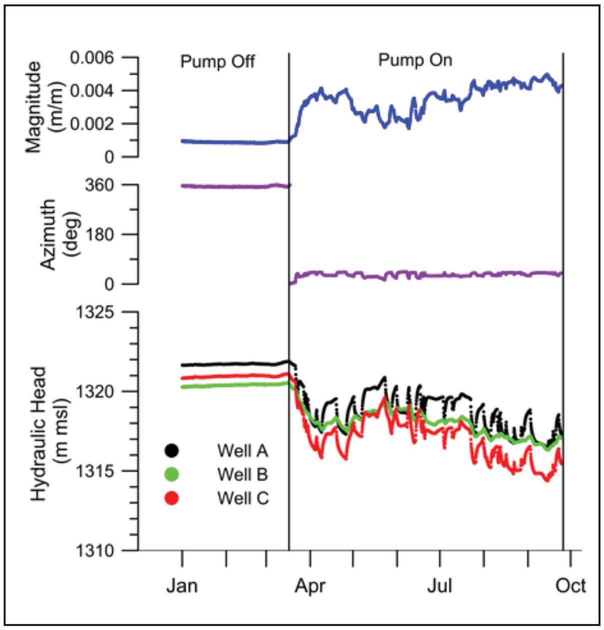

Figure 11. Hydraulic heads measured using pressure transducers in wells A, B, and C and the resulting magnitude and direction (azimuth) of the hydraulic gradient measured in degrees from North. When the irrigation well is not pumping, the hydraulic gradient is low and oriented approximately north. During the agricultural season, the hydraulic gradient increases in magnitude and the orientation shifts to the northeast toward the irrigation well.

4.2 Application #2: Characterization of Groundwater/Surface Water Interactions

Groundwater/surface water interactions can control the transport and the environmental fate of dissolved constituents at sites where surface water bodies are hydraulically connected with the aquifer. The following example illustrates the use of the 3PE spreadsheet in the initial hydrologic characterization of these interactions.

4.2.1 Background and Setting

The site is a closed waste management unit (WMU) situated adjacent to a pond. The site lies above an unconsolidated alluvial aquifer that is approximately 30 m thick. Estimates of average hydraulic conductivity within the aquifer range from approximately 3 m/d to over 15 m/d. In general, groundwater flows north beneath the WMU and has elevated concentrations of inorganic constituents. Based on an initial potentiometric surface map, it appears that groundwater on the eastern margin of the site may discharge to the pond.

4.2.2 Characterization Objective

The investigation was designed to answer the following question:

Do hydraulic gradients near the pond provide consistent indications of groundwater flow toward the pond or do they fluctuate through time?

4.2.3 Network Design and Data Acquisition

For the initial investigation of temporal trends in hydraulic gradients, hydraulic head was monitored in three existing wells near the portion of the pond where groundwater discharge was indicated in the potentiometric surface. Surface water elevation was also monitored using a staff gauge installed in the pond. Monitoring was performed for ten months to evaluate potential seasonal differences in hydraulic gradients using pressure transducers programmed to obtain a synchronous data set twelve times each day and periodic manual measurements of groundwater elevations to confirm the accuracy of the pressure transducer data.

4.2.4 Results and Conclusions

The magnitude and direction of the hydraulic gradient were calculated using 3PE for each of the data sets obtained from the wells during the monitoring period (Figure 12). In addition, precipitation data were obtained from a nearby weather station to aid in conceptualizing site conditions. The results indicate that the direction of the hydraulic gradient was generally toward the pond (Figure 13) with the exception of three very brief periods when surface water elevations rose rapidly in response to rainfall and temporary increases in the height of the dam due to beaver activity. During these brief periods, the flow direction near the pond reversed and the pond provided recharge to the aquifer. The magnitude of the hydraulic gradient appeared to display some degree of seasonality with a decrease in magnitude during the late summer and early fall that may be related to periods of decreased aquifer recharge from precipitation in areas upgradient of the pond.

This initial characterization of the hydrologic setting near the pond provided the framework for subsequent investigations of the transport of dissolved constituents into the pond. Based on the results of these investigations, remedial measures were ultimately undertaken to mitigate further discharge of contaminated groundwater.

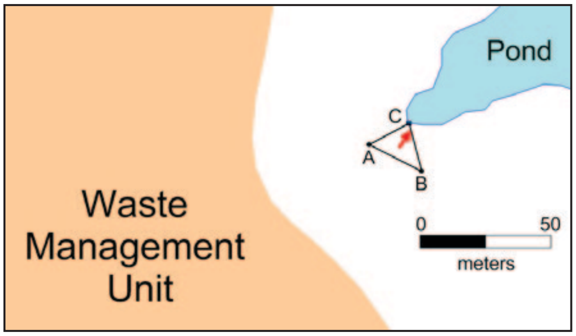

Figure 12. Site map depicting triangle of wells used to estimate hydraulic gradients. The average hydraulic gradient vector (red arrow) indicated groundwater flow was to the pond during the monitoring period.

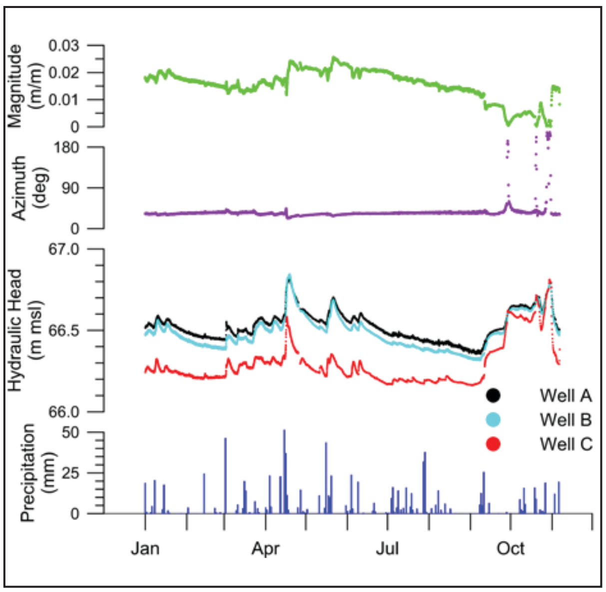

Figure 13. Hydraulic heads measured using pressure transducers in three wells and the resulting magnitude and direction (azimuth) of the hydraulic gradient measured in degrees from North. Precipitation data was obtained from a nearby meteorological station.

4.3 Application #3: Spatial and Temporal Characterization of Groundwater Flow

The determination of groundwater flow directions is often one of the first steps in characterizing the hydrogeology of a site. Potentiometric surface maps can provide a general understanding of groundwater flow directions at a given point in time. The following example depicts the use of 3PE in conjunction with software to visualize the average (net) groundwater flow directions across a site.

4.3.1 Background and Setting

The site was the location of unregulated dumping of industrial waste. The aquifer of interest consists of glacial till and outwash deposits. Hydraulic conductivity values range from 0.01 m/d to over 100 m/d. Groundwater generally flows in a northwesterly direction before discharging to a stream.

4.3.2 Characterization Objective

The investigation was designed to answer the following question:

What are the average groundwater flow directions in key areas of the site?

4.3.3 Network Design and Data Acquisition

Ten wells screened in the shallow glacial aquifer were chosen for this investigation. The wells were selected to form a triangular network of eleven, three-point problems, providing coverage across the central portion of the site. The wells were instrumented with pressure transducers which recorded changes in water levels twelve times each day for a sixteen month period. Manual water-level measurements were obtained five times during the study using an electric water-level indicator to document the accuracy of the pressure transducers and allow evaluation of transducer drift.

4.3.4 Results and Conclusions

Groundwater elevations across the site varied up to approximately 0.8 m during the study period in response to precipitation events (Figure 14). The groundwater flow directions remained relatively consistent and the magnitude of the hydraulic gradient varied less than a factor of two throughout the study, despite the significant variations in water levels (Figure 15). Note that the direction of the hydraulic gradient vector was approximately north (i.e., 0 degrees) throughout the monitoring period. The large shift in the numerical value of the azimuth from approximately 0 degrees to approximately 360 degrees merely reflects a small shift in direction from slightly east of north to slightly west of north. In this case, the net groundwater flow direction was determined by averaging the calculated hydraulic gradient direction data for each triangle in the monitoring network. The average hydraulic gradient directions and coordinates for each triangle were overlaid on a site base map. The resulting figure (Figure 16) illustrates the average directions of the hydraulic gradients and, therefore, groundwater flow during the sixteen month time period.

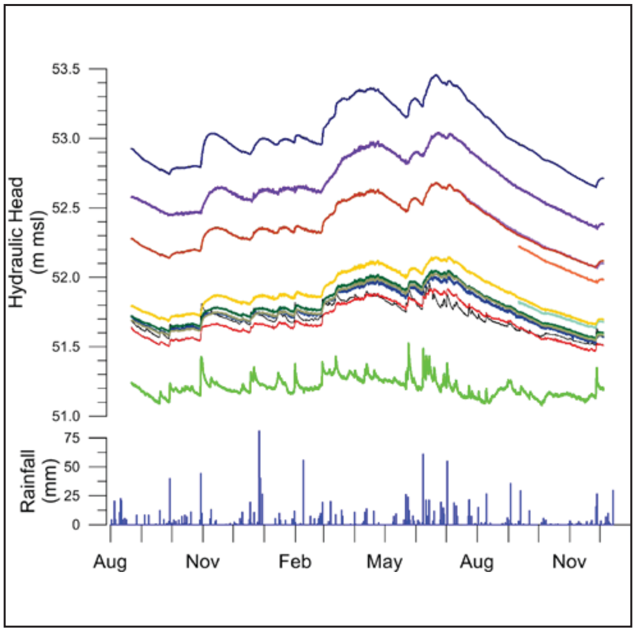

Figure 14. Hydrographs of hydraulic head in monitoring wells and precipitation measured at a local meteorological station.

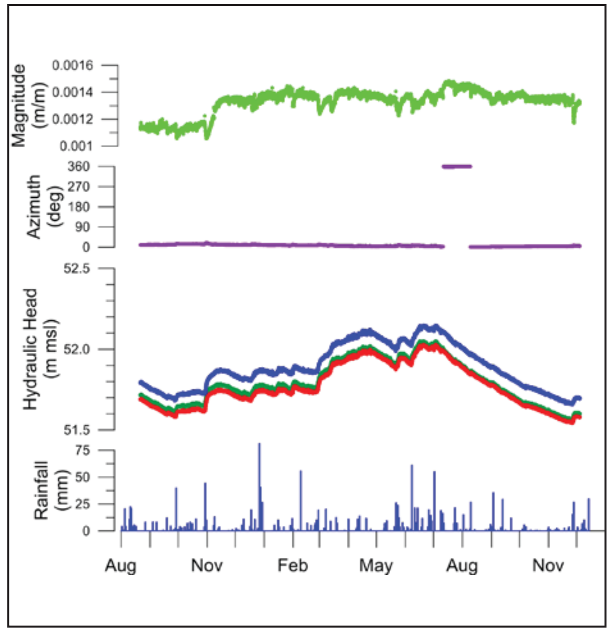

Figure 15. Hydraulic heads measured using pressure transducers in three wells and the resulting magnitude and direction (azimuth) of the hydraulic gradient measured in degrees from North. Precipitation data were obtained from a local meteorological station.

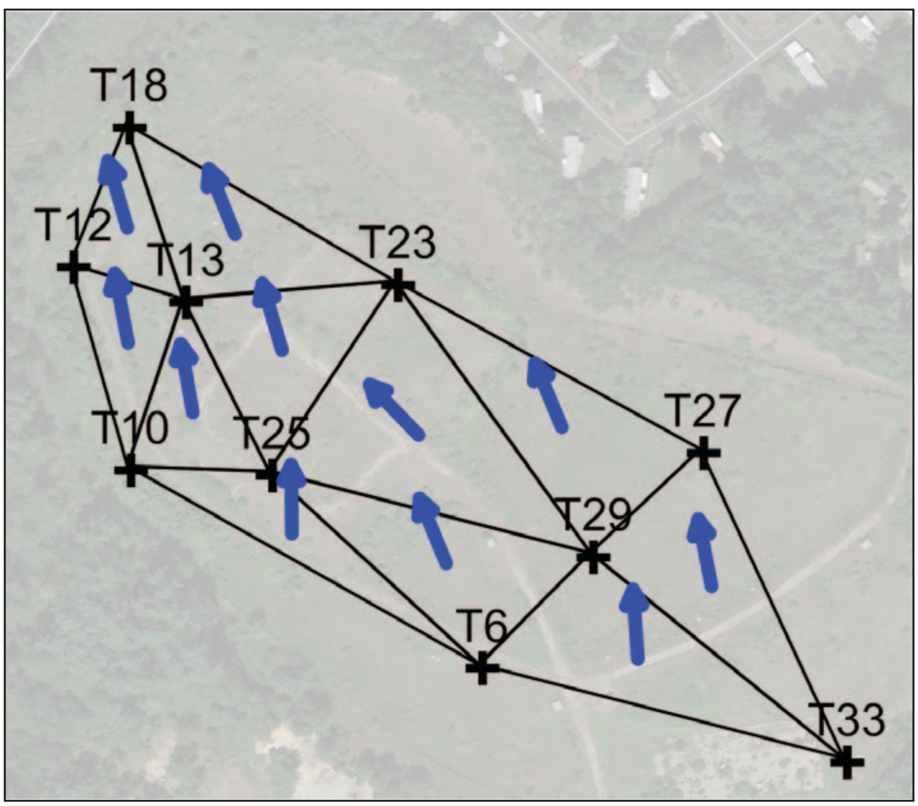

Figure 16. Network of three-point problems used to calculate hydraulic gradients and the resulting average (net) direction of the hydraulic gradient within each triangle (blue arrows). The monitoring wells (crosses), three-point solution triangles, and output vectors from the 3PE spreadsheet were plotted on a georeferenced base map to enhance conceptualization of groundwater flow directions across the site.

4.4 Application #4: Demonstration of Anisotropy Effects on Groundwater Flow

The principal direction of groundwater flow in a setting where the hydraulic conductivity varies with horizontal direction is a function of the directions and magnitudes of the minimum and maximum hydraulic conductivity axes and the direction of the hydraulic gradient. It is often instructive to illustrate the potential effects of anisotropy on groundwater flow assuming various hydraulic conductivity distributions that may be encountered in a given geologic setting. Such illustrations can aid in developing site-specific conceptual models.

4.4.1 Problem Description

For this example, the 3PE spreadsheet was used to calculate the principal direction of groundwater flow assuming isotropic and anisotropic conditions. The results were then compared to evaluate the potential effects on a site-specific conceptual model. For these calculations, data concerning horizontal anisotropy characterized by Stoner (1981) in a coal bed aquifer were used in conjunction with an assumed set of hydraulic head data to aid in visualizing the potential effects of this anisotropy on an arbitrary groundwater flow field.

4.4.2 Example Data Set

The aquifer was assumed to have an average maximum hydraulic conductivity of 0.65 m/d oriented 85 deg clockwise from North. The average minimum hydraulic conductivity is assumed to be 0.26 m/d and perpendicular to the direction of the maximum hydraulic conductivity. The following well configuration and hydraulic head data were assumed for this illustration:

Well	X Coordinate (m)	Y Coordinate (m)	Hydraulic Head (m msl)
Well A	722229	156500	100.00
Well B	722179	156400	100.00
Well C	722279	156400	99.00

4.4.3 Results and Conclusions

Under the given conditions, the direction of the hydraulic gradient, which would be the groundwater flow direction in an isotropic setting, is significantly different from the groundwater flow direction due to the moderate anisotropy in horizontal hydraulic conductivity. The groundwater flow direction shifts markedly toward the direction of maximum hydraulic conductivity. In this case, the horizontal hydraulic conductivity only varied by a factor of 2.5 (i.e., $K_{max}/K_{min} = 2.5$) in the horizontal plane of the aquifer. This resulted in an 18 deg difference in the direction of groundwater flow from the direction of the hydraulic gradient (Figure 17). If the degree of anisotropy is increased to a factor of 10 (i.e., $K_{max}/K_{min} = 10$), the difference between the directions of the hydraulic gradient and groundwater flow increases to 28 deg. This indicates the potential impact that anisotropy may have on proper conceptualization of groundwater flow under anisotropic conditions.

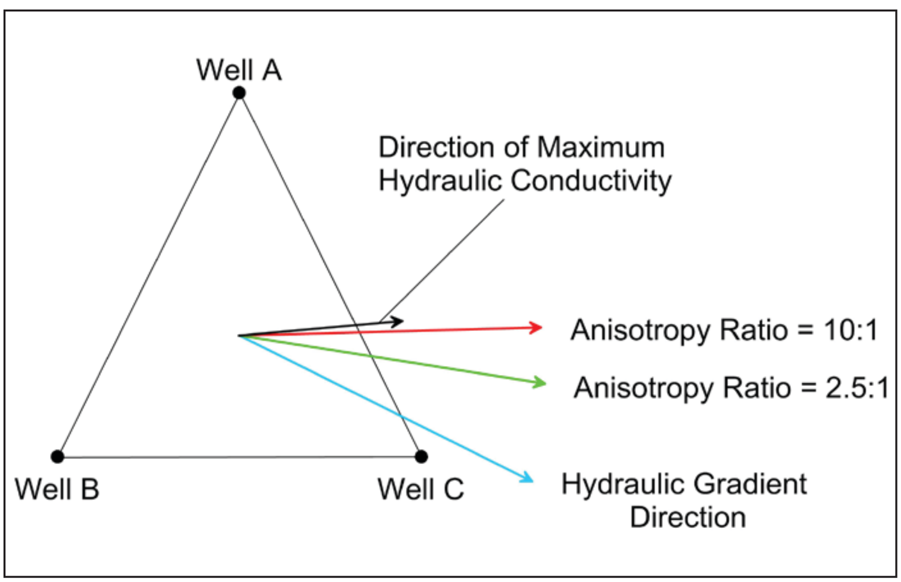

Figure 17. Effects of anisotropy in horizontal hydraulic conductivity on groundwater flow directions. The blue arrow (direction of the hydraulic gradient) would be the direction of groundwater flow in an isotropic setting. In a situation with horizontal anisotropy, the direction of groundwater flow shifts toward the direction of the maximum hydraulic conductivity as the ratio $K_{max}:K_{min}$ increases.

5.0 Summary

In many situations, hydraulic head data used in hydrogeologic assessments will increasingly be obtained using automated means, such as pressure transducers. The large amount of the data recorded can only be efficiently analyzed using a computer-based tool. The 3PE spreadsheet is a tool that can be used for solving the classical three-point problem to determine the hydraulic gradient for a single triangle or multiple triangles of monitoring points. In addition to calculating the horizontal hydraulic gradient magnitude and direction, 3PE calculates the groundwater velocity magnitude and direction, if estimates of hydraulic conductivity and effective porosity are provided.

Hydraulic gradient estimation using three-point solution methods, such as 3PE, can be a powerful tool in situations where the potentiometric surface can be represented by a plane. This tool is particularly well suited for rapid analyses of temporal trends in hydraulic gradients and groundwater flow velocity using large datasets. Potential applications for 3PE include:

- **Evaluation of the design and effectiveness of groundwater remediation systems.**

  Groundwater remediation systems require detailed knowledge of the ambient (natural) groundwater flow field for system design and the changes in the flow field during system operation. Information regarding groundwater flow rates and directions prior to system design and after remedy implementation is a critical data need for technologies using groundwater extraction or fluid injection as well as more passive technologies such as permeable reactive barriers. Similar information is also needed to design robust monitoring networks.

- **Characterization of groundwater/surface water interactions.**

  In situations where groundwater/surface water exchange may occur, initial evaluations of temporal trends in groundwater flow rates and directions can significantly improve conceptualization of the possible effects of such exchanges on surface water and, potentially, groundwater quality.

- **Rapid visualization of spatial patterns in hydraulic gradients (groundwater flow directions).**

  At sites where key portions of the groundwater flow field can be adequately represented by a series of three-point problems, 3PE offers a tool for determining groundwater flow directions across the site at individual points in time or as averages (net flow directions) that can be readily plotted on site maps for improved visualization.

- **Enhanced groundwater flow model calibration.**

  Analyses of the range of hydraulic gradients and groundwater flow directions for a number of selected triangles in sensitive areas of the model can be compared to simulated gradients and flow directions. This can provide additional confidence in the applicability of the flow model under a wider variety of hydrologic conditions.

- **Improved conceptualization of site hydrology.**

  Changes in hydraulic gradient through time can often be correlated with hydrologic changes such as local pumping or irrigation schedules, variations in aquifer recharge due to changing precipitation patterns, and changes in nearby surface water elevations. Knowledge of such correlations often leads to an improved understanding of the dominant controls on groundwater flow and appropriate engineering methods for attaining site-specific objectives.

The following general recommendations concerning the application of 3PE and other simple tools for estimating hydraulic gradients are provided:

1. Determine that the potentiometric surface in the area of interest can be adequately described as a planar surface or a series of planes.
2. Use hydraulic head data that are synchronously obtained from wells and piezometers of appropriate constructions and representative of the same portion of the aquifer.
3. Develop clear monitoring objectives. Use detailed definitions of the particular questions to be answered by the investigation to properly design the monitoring network and monitoring frequency required to support the analyses.
4. Consider potential effects of site-specific temporal variability on hydraulic gradients. Evaluation of the possible frequency of significant changes in hydraulic gradients

is the basis for informed decisions regarding the appropriate frequency for monitoring hydraulic head.

5. Consider measurement uncertainty and its effects on the attainment of investigation objectives. Monitoring network designs, measurement tools, and monitoring procedures should be chosen based, in part, on an analysis of the uncertainty that will be inherent in the measurements. In some situations, it may be useful to use 3PE to examine the sensitivity of the hydraulic gradient calculations to site-specific well placement and potential measurement errors.

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Appendix A
Theoretical Development

Figures

The solution of the three-point problem for calculations of hydraulic gradient and groundwater velocity vectors under both isotropic and anisotropic conditions implemented in the Excel® workbook relies on numerical rather than graphical methods. Mathematical conventions and notations used in the theoretical development differ slightly from those used in the spreadsheet. The conventions implemented in the spreadsheet were developed based on the hydrogeological concept of the representations of planes and angles. The derivations of the mathematical expressions implemented in Excel® are provided below.

A.1 Representing Wells in a Two-Dimensional Cartesian System

The Cartesian coordinates (Figure A-1) are unique and the points of a Cartesian plane can be identified with all possible pairs of real numbers. For example, the locations of three points on the plane are fully defined with their coordinates (i.e., (x_1, y_1), (x_2, y_2), and (x_3, y_3)).

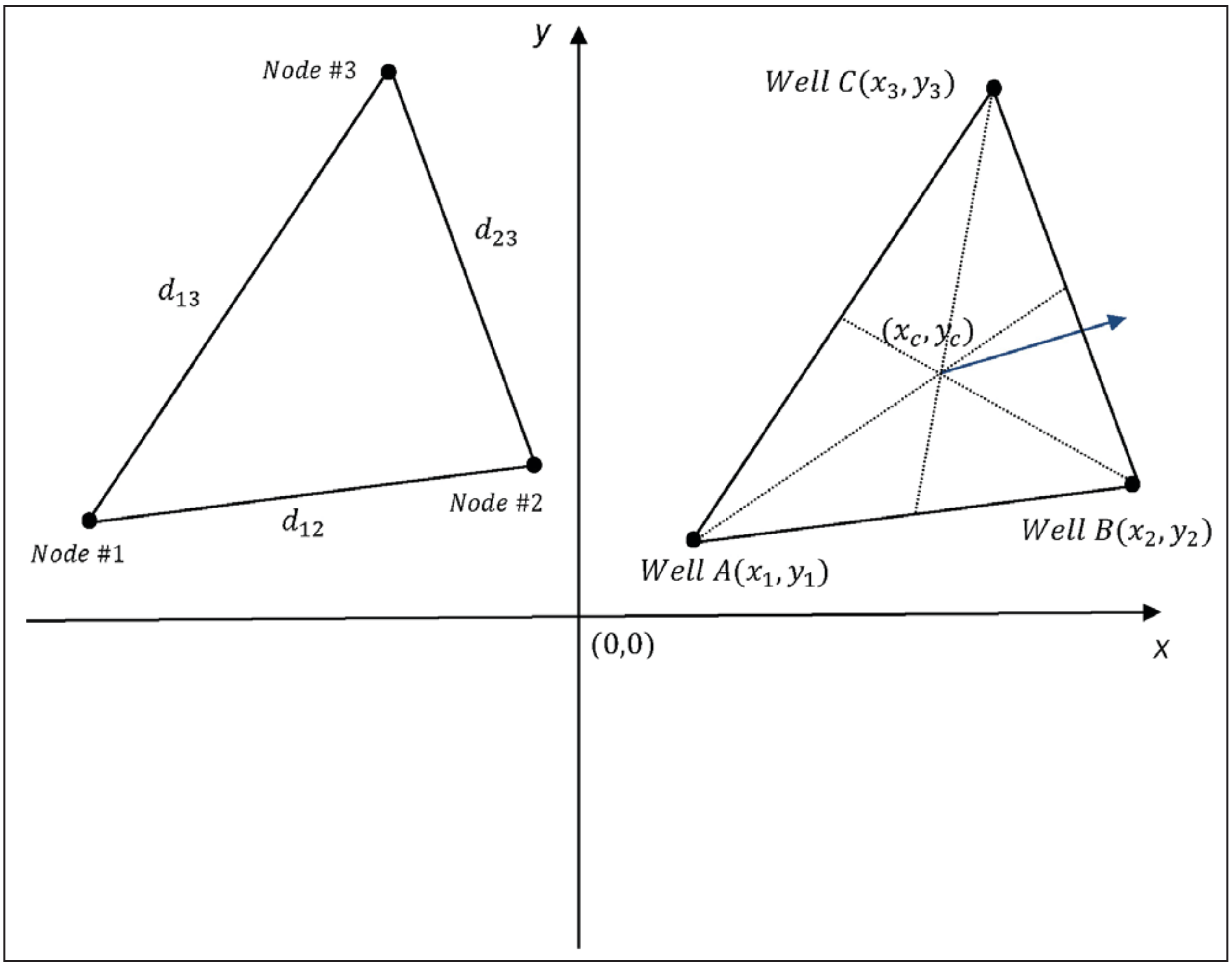


Figure A-1. Representation of the wells in a two-dimensional Cartesian system as implemented in the 3PE spreadsheet. Terminology used to designate the sides and vertices of the triangle (well locations) is depicted in the left triangle. The well coordinate convention and location of the centroid of the triangle are depicted in the triangle on the right.

The distance between the nodes (i.e., the lengths of the triangle sides) can be computed using the Cartesian version of Pythagoras' theorem:

$$d_{12} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{A-1a}$$

$$d_{23} = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2} \tag{A-1b}$$

$$d_{13} = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \tag{A-1c}$$

The *centroid* of the triangle (x_c,y_c) can be computed as

$$x_c = \frac{x_1 + x_2 + x_3}{3} \tag{A-2a}$$

$$y_c = \frac{y_1 + y_2 + y_3}{3} \tag{A-2b}$$

When the three sides of a triangle are known, the angles at the nodes (#1, #2, and #3) can be found

$$Angle\ @\#1 = \cos^{-1}\left(\frac{(d_{13})^2 + (d_{12})^2 - (d_{23})^2}{2d_{13}d_{12}}\right) \tag{A-3a}$$

$$Angle\ @\#2 = \cos^{-1}\left(\frac{(d_{12})^2 + (d_{23})^2 - (d_{13})^2}{2d_{12}d_{23}}\right) \tag{A-3b}$$

$$Angle\ @\#3 = 180 - Angle\ @\#1 - Angle\ @\#2 \tag{A-3c}$$

The area of triangle, A, is computed as

$$A = \frac{1}{2}\, det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \tag{A-4}$$

A.2 Hydraulic Gradient in 2-D Flow Field

The hydraulic gradient, ***i***, is the change in hydraulic head per unit of distance in the direction in which the maximum rate of decrease in head occurs (Heath, 1983). The direction of the slope of the water table or potentiometric surface is important because in a homogeneous and isotropic aquifer it indicates the direction of the groundwater flow.

Over a relatively small area, the water table or potentiometric surface can often be approximated as a plane. The general equation of a plane in the Cartesian coordinate system (x, y) is

$$h = Ax + By + C \tag{A-5}$$

where A, B, and C are constants of the plane equation.

Three points on the plane that form a triangle are sufficient to define the plane. That means that the hydraulic heads in three wells located at the corners of a triangle are sufficient to define the local water table or potentiometric surface. The slope of the plane (i.e., the hydraulic gradient of the water table or potentiometric surface) can be computed if the equation of the plane is known. If the corners of the triangle are labeled 1, 2, and 3 and the hydraulic heads at the corners of the triangle are $h_1 = h(x_1, y_1)$, $h_2 = h(x_2, y_2)$, $h_3 = h(x_3, y_3)$ and are known, then the following equations can be written

$$h_1 = Ax_1 + By_1 + C \tag{A-6a}$$

$$h_2 = Ax_2 + By_2 + C \tag{A-6b}$$

$$h_3 = Ax_3 + By_3 + C \tag{A-6c}$$

where A, B, and C are constants yet to be determined. The above equations (A-6a,b,c) can be written in the matrix notation as

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} \tag{A-7}$$

Solving for [A,B,C]

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \tag{A-8}$$

The hydraulic gradient, $\boldsymbol{i} = -\,\mathrm{grad}\; h = -\,\nabla h$, is a vector defined by its direction and magnitude. The components of the vector are $i_x = -\,\partial h/\partial x$ and $i_y = -\,\partial h/\partial y$. The physical interpretation of the constants A and B is that they are actually the components of the hydraulic gradient in the x- and y- direction, i_x and i_y, respectively. Inspection of Equation (A-5) reveals that the coefficient C is equal to the hydraulic head at the origin of the Cartesian system ($x = y = 0$).

The magnitude of the hydraulic gradient can now be computed from its components as

$$i = \sqrt{{i_x}^2 + {i_y}^2} \tag{A-9}$$

For the purposes of this derivation, the direction of the hydraulic gradient is the angle (α) between the positive x-axis and the hydraulic gradient (Figure A-2):

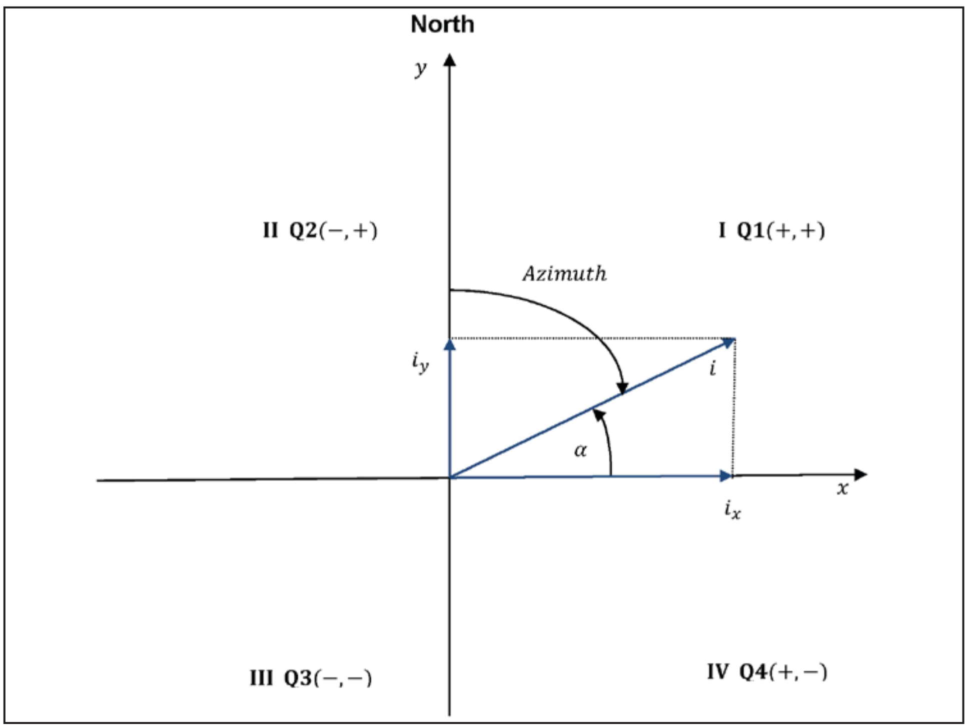


Figure A-2. Definition of the quadrants and vector angles used in this derivation. Note that the mathematical convention used to number the quadrants in the derivation proceeds in a counter-clockwise direction from the positive x-axis. In the 3PE spreadsheet, the familiar geological convention in which the quadrants are numbered in a clockwise fashion starting at the positive y-axis (North) was implemented.

$$\alpha = \tan^{-1}\left({}^{i_y}\!/_{i}\right) \tag{A-10}$$

The axes of the two-dimensional Cartesian system divide the plane into four regions called quadrants, each bounded by two half-axes. These regions are denoted by Roman numerals: **I** (+,+), **II** (-,+), **III** (-,-), and **IV** (+,-). According to the mathematical convention used in this appendix, the numbering goes counter-clockwise starting from the upper right ("northeast") quadrant (Figure A-2). Note that the convention of naming quadrants in this section differs from that used in the 3PE spreadsheet, and supporting documentation.

Equation (A-10) would yield the same angle for the case when $i_x < 0$ and $i_y > 0$ (the **II** quadrant or Q2 orientation) and in the case when the signs are reversed (the **IV** quadrant or Q4 orientation), it is important to know in which quadrant the hydraulic gradient components lie.

A.3 Specific Discharge

The specific discharge (Darcy's velocity), q, in a homogeneous *isotropic* medium can be expressed by the following equation (Bear and Verruijt, 1987, p. 34):

$$q = K\,i \tag{A-11}$$

where K is the hydraulic conductivity (L/T) and i is the hydraulic gradient (L/L).

Three-dimensional groundwater flow in a homogeneous *anisotropic* medium can be expressed by the following set of equations (Freeze and Cherry, 1979):

$$q_x = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z} \tag{A-12a}$$

$$q_y = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z} \tag{A-12b}$$

$$q_z = -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z} \tag{A-12c}$$

where q_i are the components of the specific discharge (Darcy's velocity), K_{ij} ($i, j = x, y, z$) are the components of the hydraulic conductivity tensor (a second-rank symmetric tensor), and h is the hydraulic head.

In its two-dimensional form, Equations (A-12a,b) become

$$q_x = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} \tag{A-13a}$$

$$q_y = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} \tag{A-13b}$$

The directions in space corresponding to the maximum and minimum values of the hydraulic conductivity are called the *principal directions of anisotropy*. Let a coordinate system x', y' coincide with the principal directions of anisotropy with $K_{x'} = K_{max}$ oriented along the x' axis and $K_{y'} = K_{min}$ along the y' axis (Figure A-3). Equations (A-13a,b) are then further reduced to their simplest form

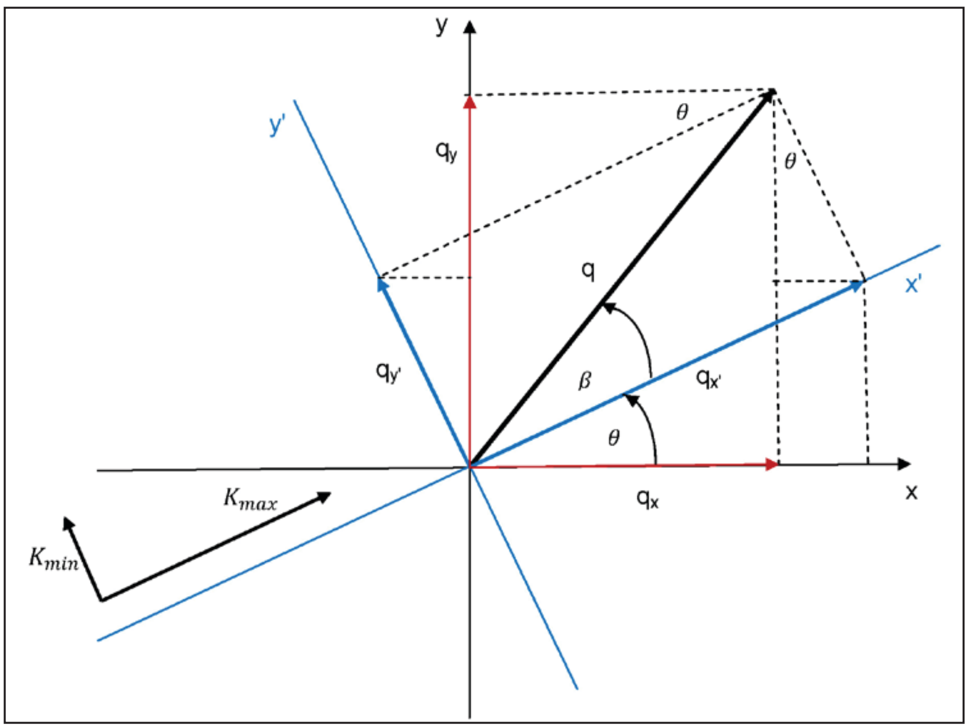


Figure A-3. Representation of the rotation of the coordinate system so that the principal directions of anisotropy, K_{max} and K_{min}, correspond with the coordinate system x', y', respectively.

$$q_{x'} = -K_{max}\frac{\partial h}{\partial x'} \tag{A-14a}$$

$$q_{y'} = -K_{min}\frac{\partial h}{\partial y'} \tag{A-14b}$$

The specific discharge q_s in the direction s is defined as

$$q_s = -K_s \frac{\partial h}{\partial s} \tag{A-15}$$

If β is the angle between the specific discharge q_s and the x'-axis (Figure A-4), then

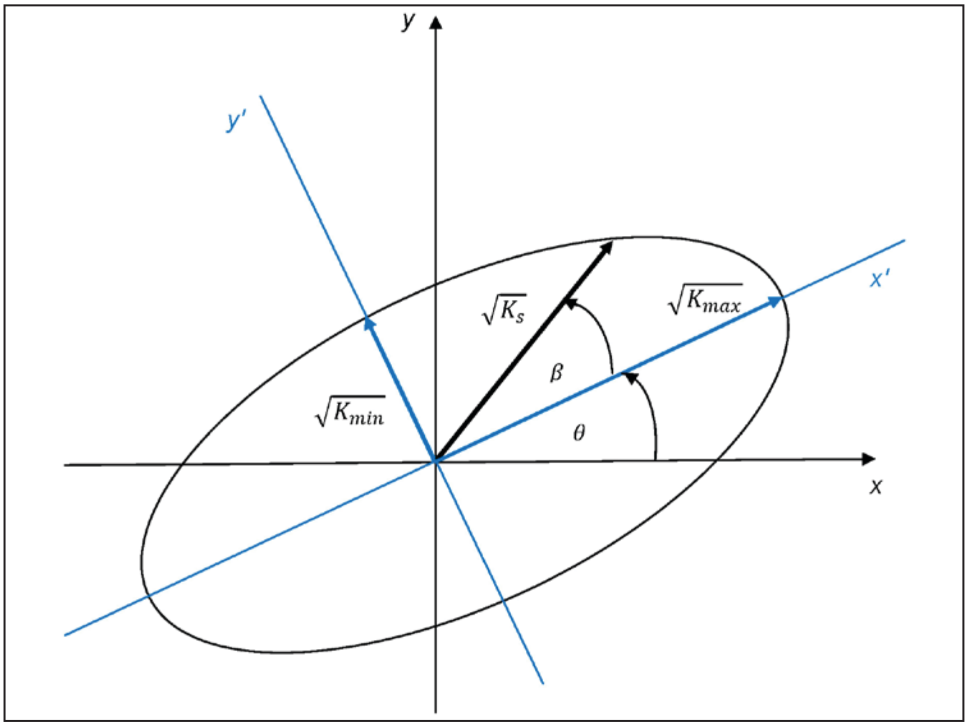


Figure A-4. Terminology used in calculation of the hydraulic conductivity ellipsoid including the major semi-axes oriented in the directions of the maximum and minimum hydraulic conductivity. The hydraulic conductivity K_s in any direction of flow in an anisotropic aquifer can be determined graphically or using Equation (A-20).

$$q_{x'} = q_s \cos\beta \tag{A-16a}$$

$$q_{y'} = q_s \sin\beta \tag{A-16b}$$

The chain rule applied to the derivatives $\partial h/\partial s$ yields

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x'}\frac{\partial x'}{\partial s} + \frac{\partial h}{\partial y'}\frac{\partial y'}{\partial s} = \frac{\partial h}{\partial x'}\cos\beta + \frac{\partial h}{\partial y'}\sin\beta \tag{A-17}$$

Substituting Equation (A-17) into (A-15) and simplifying yields

$$\frac{q_s}{K_s} = \frac{q_{x'}}{K_{max}} \cos\beta + \frac{q_{y'}}{K_{min}} \sin\beta \tag{A-18}$$

Substituting Equations (A-16a,b) into (A-17) would yield

$$\frac{q_s}{K_s} = \frac{q_s \cos\beta}{K_{max}} \cos\beta + \frac{q_s \sin\beta}{K_{min}} \sin\beta \tag{A-19}$$

Finally,

$$\frac{1}{K_s} = \frac{\cos^2\beta}{K_{max}} + \frac{\sin^2\beta}{K_{min}} \tag{A-20}$$

or since $r^2 = (x')^2 + (y')^2$, then $\cos^2\beta = {(x')^2}/{r^2}$ and $\sin^2\beta = {(y')^2}/{r^2}$

$$\frac{r^2}{K_s} = \frac{(x')^2}{K_{max}} + \frac{(y')^2}{K_{min}} \tag{A-21}$$

This equation is known as the *hydraulic conductivity ellipsoid* with major semi-axes $\sqrt{K_{max}}$ and $\sqrt{K_{min}}$ (Figure A-4). The hydraulic conductivity K_s in any direction of flow in an anisotropic aquifer can be determined graphically or using Equation (A-20) and solving it for K_s

$$K_s = \frac{K_{max}}{\cos^2\beta + \frac{K_{max}}{K_{min}} \sin^2\beta} \tag{A-22}$$

Because the coordinates of the points are often represented in a global coordinate system (e.g., UTM), which may or may not coincide with the principal directions of anisotropy, Equations (A-14a,b) in a local coordinate system cannot always be applied directly.

When the x',y' coordinate system is rotated *counterclockwise* by some angle (θ) to coincide with the x,y coordinate system, then the relationship between the specific discharge in two coordinate systems is given as

$$q_x = q_{x'} \cos\theta - q_{y'} \sin\theta \tag{A-23a}$$

$$q_y = q_{x'} \sin\theta + q_{y'} \cos\theta \tag{A-23b}$$

By substituting the specific discharge $q_{x'}$ and $q_{y'}$ into Equations (A-23a,b)

$$q_x = -K_{max} \frac{\partial h}{\partial x'} \cos\theta + K_{min} \frac{\partial h}{\partial y'} \sin\theta \tag{A-24a}$$

$$q_y = -K_{max} \frac{\partial h}{\partial x'} \sin\theta - K_{min} \frac{\partial h}{\partial y'} \cos\theta \tag{A-24b}$$

The chain rule applied to the derivatives $\partial h/\partial x'$ and $\partial h/\partial y'$ yields

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial x'} = +\frac{\partial h}{\partial x} \cos\theta + \frac{\partial h}{\partial y} \sin\theta \tag{A-25a}$$

$$\frac{\partial h}{\partial y'} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial y'} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial y'} = -\frac{\partial h}{\partial x} \sin\theta + \frac{\partial h}{\partial y}\cos\theta \tag{A-25b}$$

Substituting Equations (A-25a,b) into (A-24a,b) yields

$$q_x = -K_{max}\left(\frac{\partial h}{\partial x} \cos\theta + \frac{\partial h}{\partial y} \sin\alpha\right)\cos\theta + K_{min}(-\frac{\partial h}{\partial x} \sin\theta + \frac{\partial h}{\partial y}\cos\theta)\ \sin\theta \tag{A-26a}$$

$$q_y = -K_{max}\left(\frac{\partial h}{\partial x} \cos\theta + \frac{\partial h}{\partial y} \sin\theta\right)\sin\theta - K_{min}(-\frac{\partial h}{\partial x} \sin\theta + \frac{\partial h}{\partial y}\cos\theta)\ \cos\theta \tag{A-26b}$$

By multiplying and grouping the similar terms

$$q_x = -K_{max}\frac{\partial h}{\partial x}\cos^2\theta - K_{max}\frac{\partial h}{\partial y}\sin\theta\cos\theta - K_{min}\frac{\partial h}{\partial x}\sin^2\theta + K_{min}\frac{\partial h}{\partial y}\sin\theta\cos\theta \tag{A-27a}$$

$$q_y = -K_{max}\frac{\partial h}{\partial x}\sin\theta\cos\theta - K_{max}\frac{\partial h}{\partial y}\sin^2\theta + K_{min}\frac{\partial h}{\partial x}\sin\theta\cos\theta - K_{min}\frac{\partial h}{\partial y}\cos^2\theta \tag{A-27b}$$

Simplifying,

$$q_x = -(K_{max}\cos^2\theta + K_{min}\sin^2\theta)\frac{\partial h}{\partial x} - ((K_{max} - K_{min})\sin\theta\cos\theta)\frac{\partial h}{\partial y} \tag{A-28a}$$

$$q_y = -((K_{max} - K_{min})\sin\theta\cos\theta)\frac{\partial h}{\partial x} - (K_{max}\sin^2\theta + K_{min}\cos^2\theta)\frac{\partial h}{\partial y} \tag{A-28b}$$

A comparison of Equations (A-13a,b) and (A-28a,b) provides equations for computing K_{ij} components (Strack, 1989; p. 14):

$$K_{xx} = K_{max}\cos^2\theta + K_{min}\sin^2\theta \tag{A-29a}$$

$$K_{yy} = K_{max}\sin^2\theta + K_{min}\cos^2\theta \tag{A-29b}$$

$$K_{xy} = K_{yx} = (K_{max} - K_{min})\sin\theta\cos\theta \tag{A-29c}$$

Using the trigonometric identities,

$$2\sin^2\theta = 1 - \cos 2\theta \tag{A-30a}$$

$$2\cos^2\theta = 1 + \cos 2\theta \tag{A-30b}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{A-30c}$$

the hydraulic conductivities from Equations (A-29a,b,c) can be expressed as (Bear, 1979; p.73)

$$K_{xx} = \frac{K_{max} + K_{min}}{2} + \frac{K_{max} - K_{min}}{2} \cos 2\theta \tag{A-31a}$$

$$K_{yy} = \frac{K_{max} + K_{min}}{2} - \frac{K_{max} - K_{min}}{2} \cos 2\theta \tag{A-31b}$$

$$K_{xy} = K_{yx} = \frac{K_{max} - K_{min}}{2} \sin 2\theta \tag{A-31c}$$

Equation (A-31c) can be rearranged when the right-hand side of the equation is multiplied by cos *2θ* / cos *2θ*

$$K_{xy} = \frac{K_{max} - K_{min}}{2} \tan 2\theta \cos 2\theta \tag{A-32}$$

and solved for tan *2θ*

$$\tan 2\theta = \frac{2\, K_{xy}}{(K_{max} - K_{min}) \cos 2\theta} \tag{A-33}$$

Subtracting Equation (A-31b) from Equation (A-31a) yields

$$K_{xx} - K_{yy} = (K_{max} - K_{min}) \cos 2\theta \tag{A-34}$$

Inspection of Equations (A-34) and (A-33) reveals the equation for computing the rotation angle θ when the K components are known (Bear, 1972; p.140)

$$\tan 2\theta = \frac{2\, K_{xy}}{K_{xx} - K_{yy}} \tag{A-35}$$

If the x', y' are principal directions (Figure A-3), the rotation angle θ can be computed from Equation (A-35) and the principal hydraulic conductivity components K_{max} and K_{min} are computed as (Bear, 1972; p. 141)

$$K_{max} = \frac{K_{xx} + K_{yy}}{2} + \sqrt{\left(\frac{K_{xx} - K_{yy}}{2}\right)^2 + K_{xy}{}^2} \tag{A-36a}$$

$$K_{min} = \frac{K_{xx} + K_{yy}}{2} - \sqrt{\left(\frac{K_{xx} - K_{yy}}{2}\right)^2 + K_{xy}{}^2} \tag{A-36b}$$

A.4 Groundwater Velocity

Darcy's law for groundwater flow in an *isotropic* medium can be expressed by the following equation (Freeze and Cherry, 1979):

$$V = \frac{q}{n_e} = \frac{K}{n_e} i \tag{A-37}$$

where V is the groundwater velocity, q is the specific discharge, K is the hydraulic conductivity, n_e is the effective porosity, and i is the hydraulic gradient.

In an *anisotropic* medium, the groundwater velocity components are computed as (Pinder, et al., 1981)

$$V_x = -\frac{1}{n_e}\left(K_{xx}\frac{\partial h}{\partial x} + K_{xy}\frac{\partial h}{\partial y}\right) \tag{A-38a}$$

$$V_y = -\frac{1}{n_e}\left(K_{yx}\frac{\partial h}{\partial x} + K_{yy}\frac{\partial h}{\partial y}\right) \tag{A-38b}$$

The magnitude of the groundwater velocity is computed as:

$$V = \sqrt{{V_x}^2 + {V_y}^2} \tag{A-39}$$

Using the conventions of this derivation, the direction of the groundwater velocity vector is the angle (η) between the positive x-axis and the hydraulic gradient (Figure A-5):

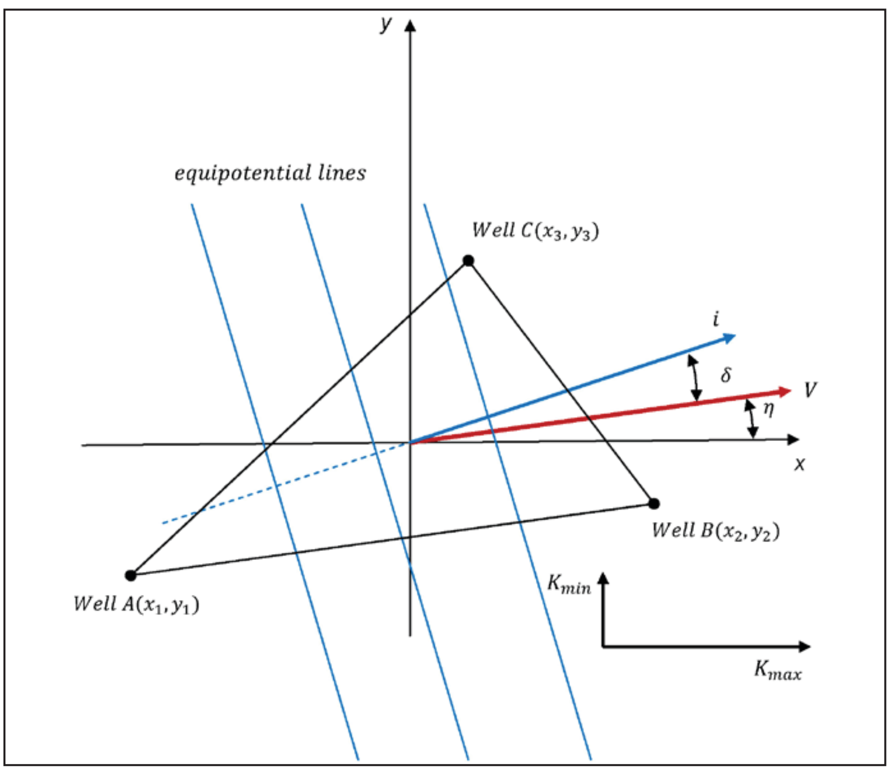


Figure A-5. Terminology used in this derivation of the direction of the groundwater velocity vector in an anisotropic medium. The hydraulic gradient vector is the blue arrow labeled ***i*** and the groundwater velocity vector is the red arrow labeled ***V***.

$$\eta = \tan^{-1}\left({V_y}/{V_x} \right) \tag{A-40}$$

In an anisotropic aquifer, the groundwater velocity and the hydraulic gradient vectors are not collinear (Figure A-5). The angle between the vectors (δ) can be computed by subtracting the *i* direction from the *V* direction; however, a better approach is to use the dot product or the inner product because the angle between the vectors is always the inner angle, which is less than 180 degrees (Fienen, 2005)

$$\delta = \cos^{-1}\left(\frac{V_x\, i_x + V_y\, i_y}{V\, i} \right) \tag{A-41}$$

The above derivation provides mathematical expressions for hydraulic gradient and groundwater velocity that can be computed using standard functions available in commercial spreadsheet software.

Appendix B
Comparison of 3PE Results with Published Problems

Results from the numerical computations using the 3PE spreadsheet were verified through comparison with results of published problems available in the literature, and with results obtained from other published hydraulic gradient calculators.

B.1 Validation of Hydraulic Gradient Computation

Heath (1983; pp. 10-11)

This is a classic problem used to explain the graphical method of solving a three-point problem. The information about the hydraulic heads is given: Well 1 (26.26 m), Well 2 (26.20 m), and Well 3 (26.07 m). The coordinates of the wells were not provided; the distances between the wells are given: d_{12} = 165.0 m; d_{23} = 150.0 m, and d_{13} = 215.0 m. The computed hydraulic gradient is (0.13 m)/(133 m) or 0.000977.

Because the well coordinates were not provided in Heath (1983), the triangle position with respect to North is not uniquely defined. For purposes of this comparison, Well 1 was placed at the origin of the coordinate system and Well 2 on the x-axis. The coordinates of Well 3 were then computed using trigonometric functions.

Well Name	X Coordinate (m)	Y Coordinate (m)
Well 1	0.00	0.00
Well 2	165.00	0.00
Well 3	154.39	149.62

The computations of the length of the triangle sides were checked:

Triangle Centroid	106.46	49.87		
Triangle Area	12,344.03	(L^2)	Angle of Triangle (degrees) @	
Distance #1 - #2	165.00	(L)	Well 1	44.10
Distance #2 - #3	150.00	(L)	Well 2	85.95
Distance #1 - #3	215.00	(L)	Well 3	49.95

The magnitude of the hydraulic gradient computed by 3PE is 0.000966, which is equivalent to the published value considering that the Heath (1983) solution was determined graphically and, therefore, subject to the increased uncertainty associated with graphical solutions.

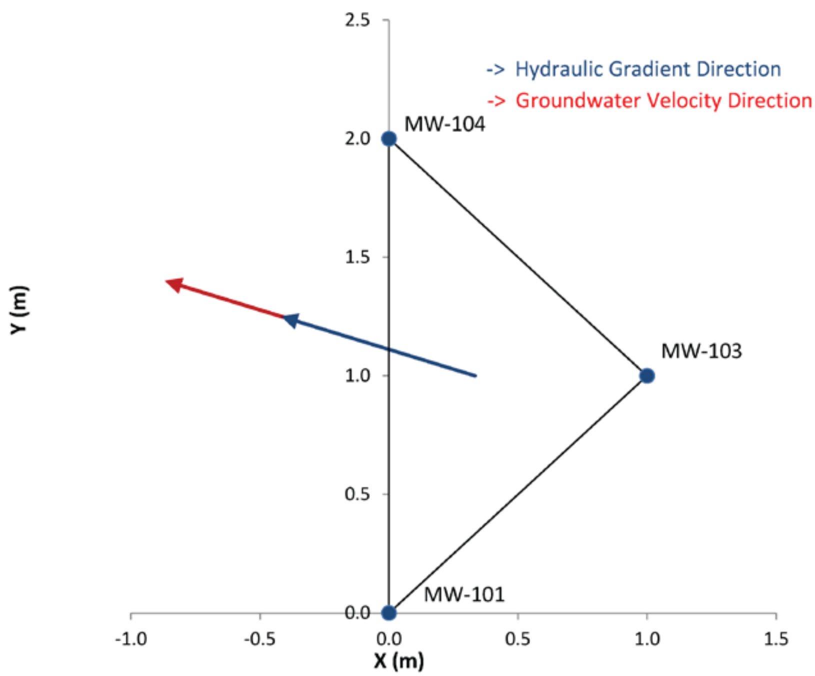
Pinder et al. (1981)

The coordinates and the hydraulic heads are given. The hydraulic heads are: Well 101 (11.00 m), Well 104 (12.00 m), and Well 103 (10.00 m) and the well coordinates are:

Well Name	X Coordinate (m)	Y Coordinate (m)
MW-101	0.00	0.00
MW-104	1.00	1.00
MW-103	0.00	2.00

The hydraulic conductivity components K_{xx} and K_{yy} are both 2 m/d and the effective porosity is 0.25. The published results are as follows: the velocity components V_x and V_y are -12 m/d and 4 m/d, respectively; the hydraulic gradient components i_x and i_y are -1.5 and 0.5, respectively.

The 3PE results are identical to the published results. 3PE also computed the orientation of the velocity vector as 288 degrees.

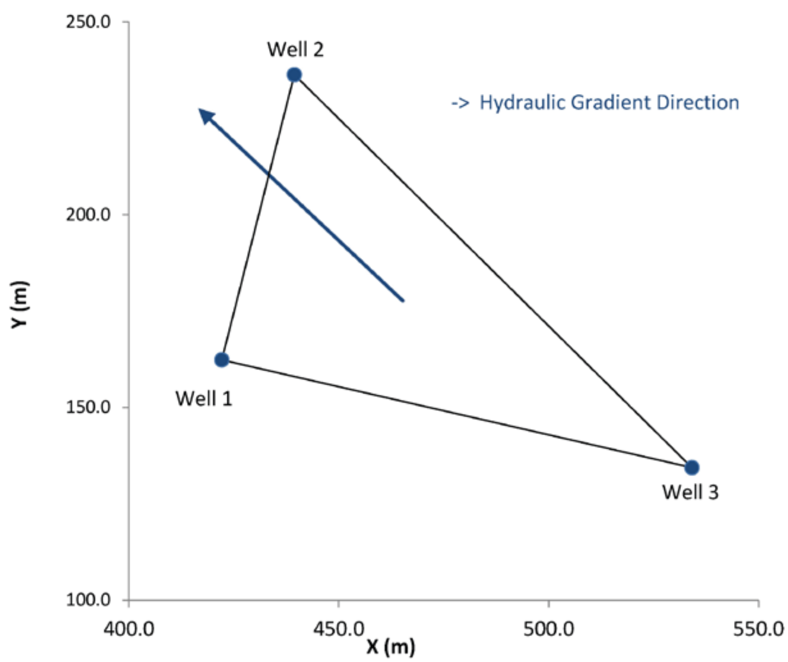


Silliman and Frost (1998)

The hydraulic heads and well coordinates are given. The hydraulic heads in Well 1, Well 2, and Well 3 are 132.37 m, 131.86 m, and 132.01 m, respectively. The well coordinates are as follows:

Well Name	X Coordinate (m)	Y Coordinate (m)
Well 1	534.12	134.37
Well 2	439.43	236.34
Well 3	422.13	162.33

The magnitude of the gradient is 0.003667 (m/m) and the orientation is reported to be "approximately 45 degrees below the positive x-axis" (Silliman and Frost, 1998, p. 518). The magnitude of the gradient computed by 3PE has the same value. However, the orientation of the hydraulic gradient vector computed by 3PE is 316 degrees in the clockwise direction from North, which is different from the reported orientation. The orientation computed by 3PE was verified using a graphical solution. It appears that the published orientation is reported incorrectly, and should read "approximately 45 degrees above the negative x-axis".



On-Site: the On-line Site Assessment Tool - Three Point Gradient Calculator (USEPA, 2014)

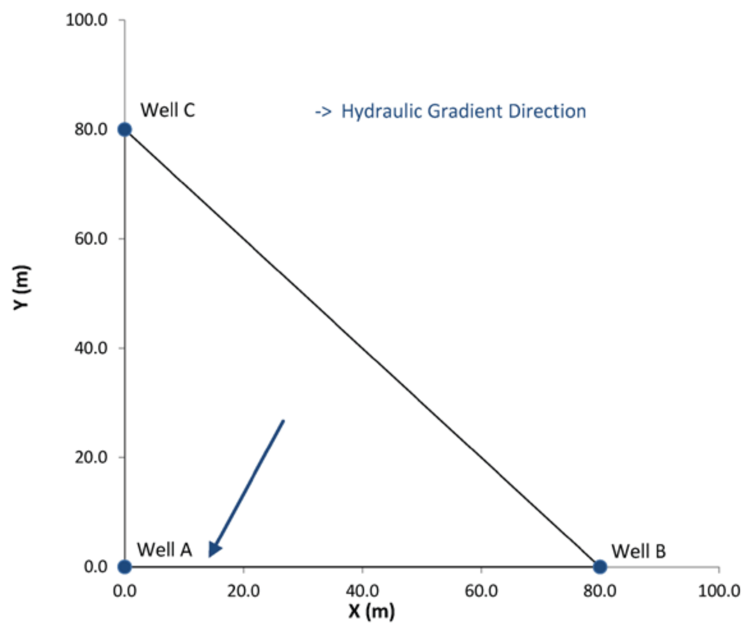
The output from the 3PE spreadsheet was compared with the results of a sample problem computed using the on-line Three Point Gradient calculator (USEPA, 2014). For this example, the well coordinates and hydraulic heads presented in Silliman and Frost (1998) and discussed above were used as input into the on-line calculator. The magnitude of the gradient computed using the on-line calculator is 0.003666 and the orientation is 316 degrees from North, which is the same as computed using the 3PE spreadsheet.

Gradient.XLS (Devlin, 2003)

The output from the 3PE spreadsheet was also compared with the results of two sample problems computed using the spreadsheet Gradient.XLS (Devlin, 2003). For the first problem, the well coordinates and hydraulic heads are as follows:

Well Name	X Coordinate (m)	Y Coordinate (m)	Hydraulic Head (m)
Well A	0.00	0.00	100.0
Well B	80.00	0.00	101.0
Well C	0.00	80.00	102.0

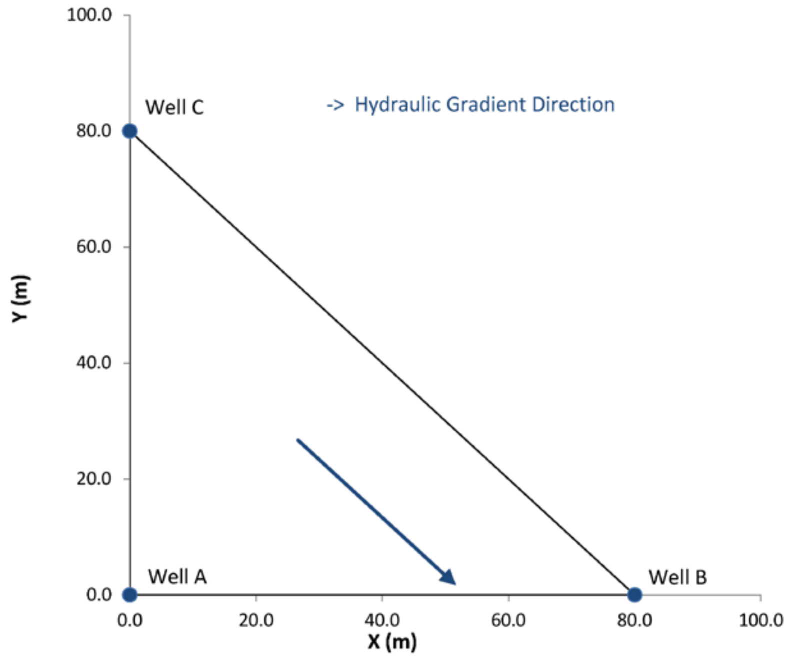
The magnitude of the hydraulic gradient vector computed using Gradient.XLS is 0.02795 with an orientation of -116.6 degrees off the x-axis or an azimuth of 206.6 degrees from North. Identical results were obtained using 3PE.



For the second problem, the well coordinates were the same as in the first comparison but the hydraulic head data were:

Well Name	Hydraulic Head (m)
Well A	101.0
Well B	100.0
Well C	102.0

The magnitude of the hydraulic gradient vector computed using Gradient.XLS is 0.01768 with an orientation of -45.0 degrees off the x-axis or an azimuth of 135.0 degrees from North. Identical results were again obtained using 3PE.



B.2 Validation of Velocity Computations

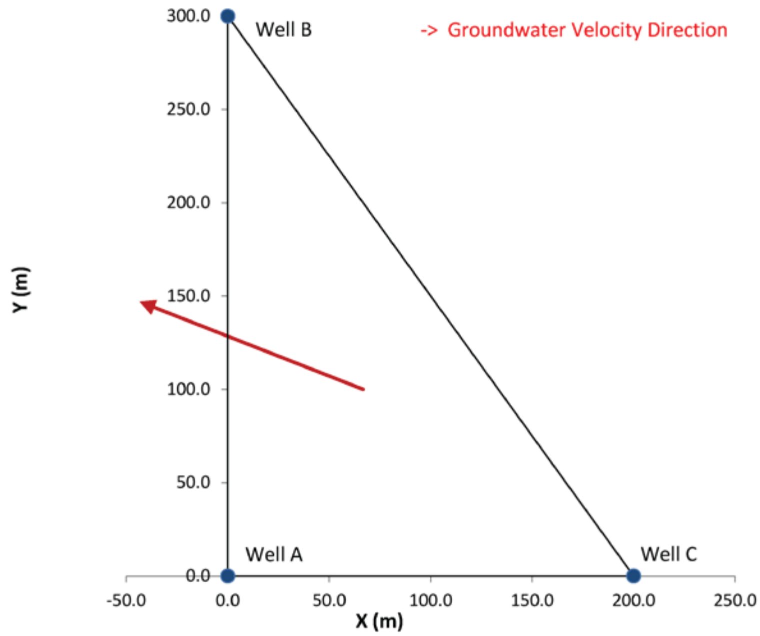
Bear, J. and A. Verruijt (1987) Problem 2.7 (p. 387)

This problem is used to validate the computation of the direction of the groundwater velocity vector in an isotropic aquifer.

- Given the piezometric heads in three observation wells located in a homogeneous confined aquifer of constant transmissivity $T = 5{,}000\ m^2/d$ (Well A (0,0, h = 10.0 m), Well B (0, 300, h = 8.4 m) and Well C (200, 0, h = 12.5 m)), determine the direction of discharge through the aquifer.

Well Name	X Coordinate (m)	Y Coordinate (m)
Well A	0.00	0.00
Well B	0.00	300.00
Well C	200.00	0.00

- ANSWER: The direction of discharge through the aquifer is -23 degrees from the negative x-axis.



The orientation of the groundwater flow velocity vector calculated by 3PE is 293 degrees (23 degrees clockwise from the negative x-axis), which is identical to the published answer.

Bear, J. and A. Verruijt (1987) Problem 2.9 (p. 387)

The following problem is used to validate the computation of the orientation of the groundwater velocity vector in an anisotropic aquifer.

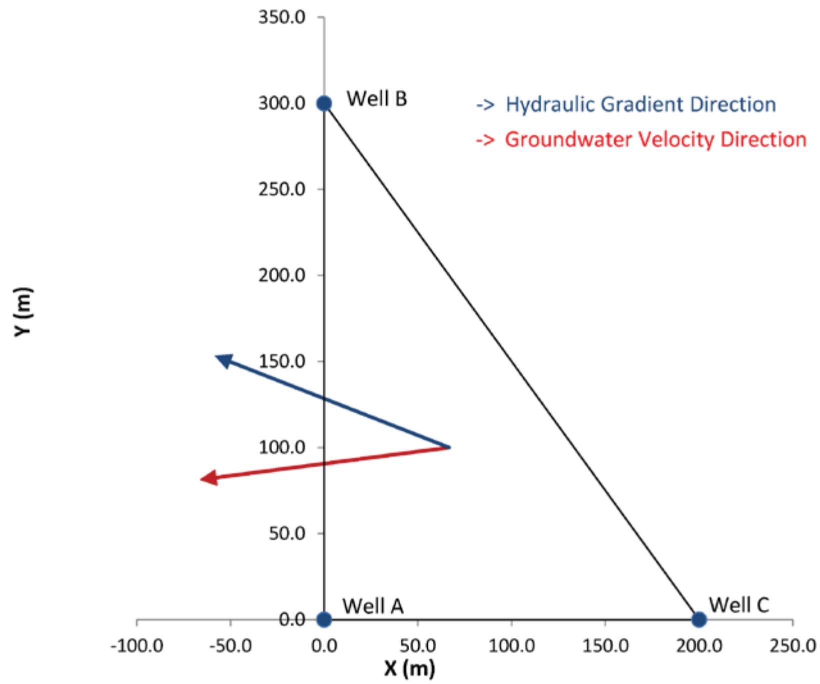
- Repeat Problem 2.7 when the aquifer is anisotropic with K_{xx} = 30 m/d, K_{yy} = 10 m/d, and $K_{xy} = K_{yx}$ = 8 m/d. The aquifer thickness is 50 m.

- ANSWER: The discharge direction is -8 degrees from the negative x-axis.

In order to use 3PE in the solution of this problem, the principal hydraulic conductivity components and the angle of rotation must be calculated for input into the spreadsheet. The angle of rotation is computed using Equation A-35 and the principal hydraulic conductivity components are computed with Equations A-36a,b:

Principal Hydraulic Conductivity Components		
Kmax =	32.81	(L/T)
Kmin =	7.19	(L/T)
Orientation of Kmax =	70.67	(degrees from N)
θ =	19.33	(degrees from X)

The orientation of the groundwater velocity vector calculated by 3PE is 262 deg clockwise from North (or 8 degrees measured counter-clockwise from the negative x-axis), which is identical to the vector orientation given in the published problem.



Bear, J. and A. Verruijt (1987) Problem 2.11 (p. 387)

The following problem is used to validate the computation of the angle between the groundwater velocity vector and the hydraulic gradient vector in an anisotropic aquifer.

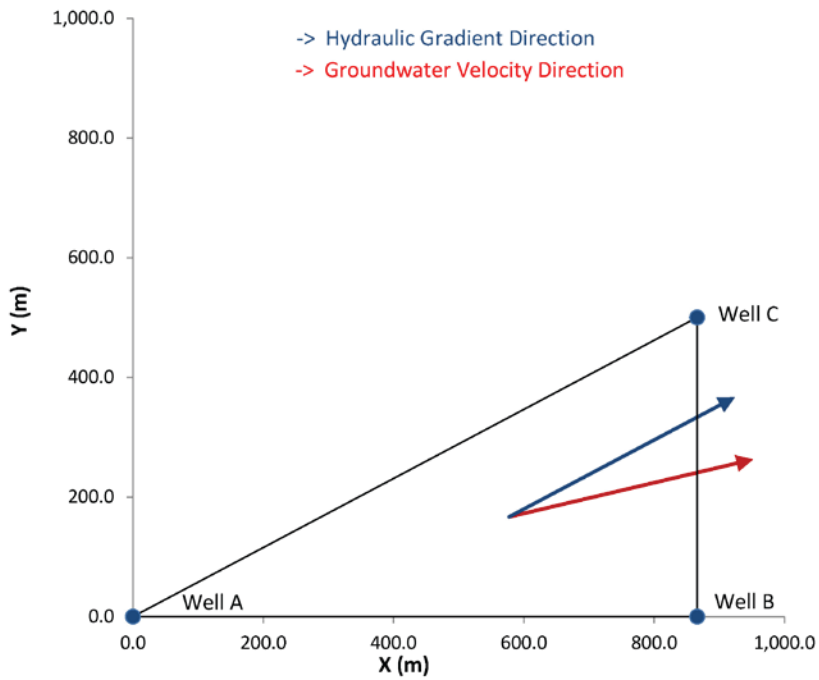
- Let K_x = 36 m/d and K_y = 16 m/d be the principal values of hydraulic conductivity in an anisotropic aquifer, in the *x* and *y* directions, respectively, in two-dimensional flow. The hydraulic gradient is 0.004 in a direction making an angle of 30 degrees with the positive *x*-axis. Determine the angle between the vectors.

- ANSWER: 15.6 degrees.

Because the well coordinates and the hydraulic heads were not given, a simple layout of the wells was assumed and the hydraulic heads were computed based on the given hydraulic gradient.

Well Name	X Coordinate (m)	Y Coordinate (m)	Hydraulic Head (m)
Well A	0.00	0.00	104.0
Well B	866.00	0.00	101.0
Well C	866.00	500.00	100.0

The direction of the hydraulic gradient calculated by 3PE is 60 degrees (30 degrees counter-clockwise from the positive *x*-axis); and the direction for the velocity vector is 76 degrees (14 degrees counter clockwise from the positive *x*-axis). The difference between the directions of the hydraulic gradient vector and velocity vector is 16 degrees, in close agreement with the published result.



Abriola and Pinder (1982)

The following problem is used to validate the computation of the magnitude of velocity vector components in an isotropic aquifer.

- The horizontal velocity components are calculated for wells with coordinates and hydraulic heads given as: Well 1 (-10, 10, 20.0), Well 2 (0, 0, 20.0) and Well 4 (0, 10, 22.8). Also given are values for porosity (0.25) and $K_{xx} = K_{yy} = 2$.
- The published results are V_x = -2.24 and V_y = -2.24.

The 3PE results are identical to the published results (V_x = -2.24 and V_y = -2.24).

Appendix C
3PE Workbook

Excel® file available for download at http://www.epa.gov/nrmrl/gwerd/publications.html

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