Modeling Particulate Charging in ESPs

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Abstract—**In electrostatic precipitators there is a strong interaction between the particulate space charge and the operating voltage and current of an electrical section. Calculating either the space charge or the operating point when the other is fixed is not difficult, but calculating both self-consistently is much more difficult. A method is proposed that makes the problem straightforward. An iterative solution is required, but the closure rate should be acceptable.**

Introduction

ALTHOUGH adequate models exist for fine-particle charging in dilute aerosols, the charging situation in electrostatic precipitators (ESPs) requires a different approach. Because of the high particulate density and space charge suppression of corona, the ESP section is not easily described with standard charging models.

Usually the particulate space charge is large enough to raise the operating voltage of an inlet electrical section by several kilovolts and, at the same time, reduce the average current in the section to less than half the value in the succeeding sections. A correlation illustrating this point is taken from [1], where a sample of ESP operating points was fitted on a section-by-section basis, in terms of the average electric fields and current densities in each ESP.

The derived E–J(V–I) curves for each section are shown in Fig. 1, where the offset of corona onset voltage is shown clearly at low current density. If we use the E–J curve of the fourth section as a reference, by assuming a low value for the average space charge, then the average space charge in each prior section can be estimated from the change in corona onset voltage by the equation

$$\Delta V = S/2\epsilon \cdot b^2 \tag{1}$$

where ΔV is the change in voltage, S is the average space charge, ϵ is the free-space permittivity, and b is the wire-plate separation. Assuming a typical value of b, 11.4 cm, the particulate space for each section is shown in Table I.

If the operating point is determined by sparking (the usual mode of operation), then the average electric field is roughly constant from section to section, and the corresponding current densities in each section increase from the first to the last.

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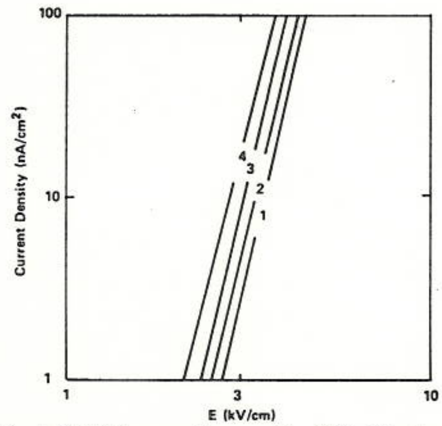

Fig. 1. Derived $EJ(V$–$I)$ curves for operating ESPs. Numbers give section location, with 1 the inlet section.

TABLE I
CALCULATED PARTICULATE SPACE CHARGE

Section	Space Charge (C/m^3)
1	8.1E−06
2	5.9E−06
3	4.4E−06
4	1.5E−07 (assumed)

These particulate space charge densities are comparable to the ionic space charge densities at normal current levels, typically in the range 1–4 μC/m^3. Thus it is to be expected that the particulate space charge and ionic space charge would have comparable effects on the electrical conditions.

In standard charging theories [2], [3] implicit assumptions are made that all particles are exposed to an invariant supply of ions. The charging rate equations can then be integrated to obtain the charge as a function of time. In the ESP situation, however, the charge acquired by the particles collectively affects the supply of ions for further charging.

It would be useful to avoid a numerical integration of the rate over the charging zones of the ESP. When numerical solutions of Poisson's equation are used with particulate and ionic space charge, almost any degree of accuracy can be achieved, but at a great cost in time. If the complication of adding an incremental particle charge calculation is included, the computations become prohibitive.

In this paper an alternative calculation method will be described which aims at an iterative approach to an "equilibrium" between the charging current and the resulting space charge on the particles. The particulate space charge is made

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the parameter of interest because its value can be bounded for a given ESP. Closure with standard charging and collection models is obtained by finding appropriate average values for particle charge and mobility, as functions of position along the direction of flow, and then by iterating the process.

ESP *V–I* Curves with Space Charge

A useful approximation of the voltage–current relation was given by Cooperman [4] in a form that has been applied to many corona devices:

$$j = KV(V - VO), \tag{2}$$

where j is the average current density, V is the applied voltage, VO is the corona start voltage, and K is constant with voltage given by

$$K = 2\mu\epsilon / b^3, \tag{3}$$

where μ is the ion mobility, ϵ is the free-space permittivity, and b is the wire–plate separation. This form is in fair agreement with a better approximation [5], but the differences do not matter in the present situation. When a constant space charge is added to the ESP, the corona onset voltage, VO, is raised by the amount given by (1). The voltage offset as a function of average space charge is shown in Fig. 2, where the orientation of the figure is chosen for later use.

From [5] it is found that the constant K also varies with the space charge (fundamentally caused by the increased electric field) as

$$K = (2\mu\epsilon / b^3) \cdot (1 + S/4\epsilon b), \tag{4}$$

where all terms are as previously defined.

This combination of effects results in a typical set of V–j curves for different space charge values, shown in Fig. 3. The curves in Fig. 3 are very similar to the correlation curves in Fig. 1, including the change of slope with increasing space charge.

For an ESP operating at a fixed voltage, shown as a vertical line in Fig. 3, there is a definite range of space charge for which current can flow—from zero to a maximum value. The maximum space charge at which ΔV is equal to the difference between the operating voltage and the clean gas corona start voltage is

$$S_{\max} = 2\epsilon / b^2 \cdot (Vop - VO), \tag{5}$$

where VO is understood to be the clean-gas starting voltage. It is possible for S to exceed this value, for instance, with a precharger preceding an ESP section; but if the section is to supply the charging current, Smax is the largest possible average value.

For the sake of further development, we assume that particulate space charge (calculated from charging of individual particles) is spatially uniform in the direction from wire to plate. This simplification may not be realistic for high particle concentrations but permits a more succinct treatment. For the same reason we assume that the value of the space charge is constant within the region serviced by a single wire, although it may vary from wire to wire.

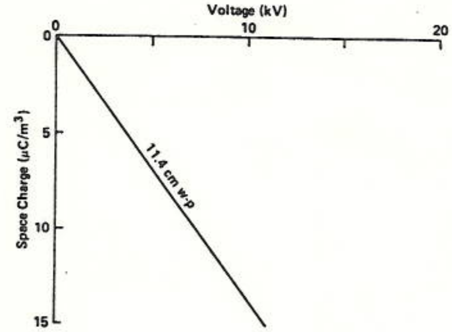

Fig. 2. Voltage offset caused by space charge, shown here for typical wire–plate spacing.

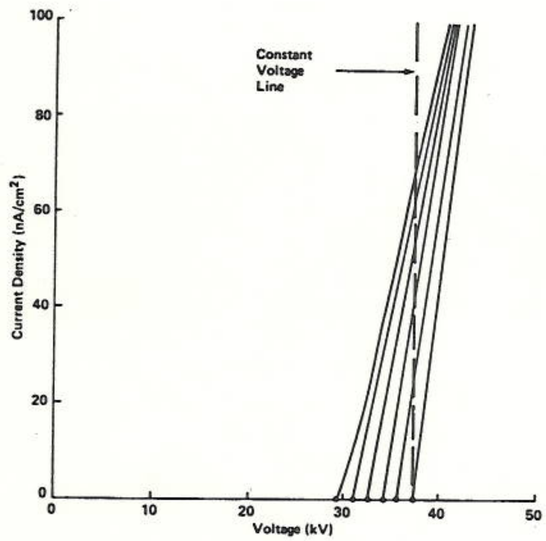

Fig. 3. Theoretical V–j curves with space charge increasing from 0 to 10 $\mu C/m^3$.

Even though no ions may reach the plate from the wire (as, for instance, in space-charge quenching, where all the ions are absorbed on the particles), there are two real currents that may be observed with particulate space charge: the particle charging current and the particle collection current. The first results from the ions attaching to particles, and the second from particles being collected. These currents represent minimum values of the operating current and should be accounted for.

Ionic and Particulate Space Charge

Although ionic and particulate space charges affect the divergence of the electric field identically, the difference in mobility affects the V–L curve in different ways.

Sigmond [6] derives a formula for the density of ions drifting along a field line without diffusion:

$$1/\rho(t) - 1/\rho o = \mu t / \epsilon \tag{6}$$

where $\rho(t)$ is the ion density at any time after a time zero when it was ρo. That is, the ion density along a field line is determined only by the age of the ions. This result comes from the continuity equation and the current conservation equations, leading to the differential equation:

$$d\rho/dt = -\mu\rho\nabla \cdot \bar{E}. \tag{7}$$

If the divergence of $\bar{E}$ includes immobile space charge along the field line, the resulting formula is

$$[(S/\rho)+1] = [(S/\rho o)+1] \exp(\mu t S/\epsilon) \tag{8}$$

where all terms are as defined before. This formula does reduce to 6 as S approaches zero.

The usefulness of these formulas lies in their ability to give good approximate answers when the ion transit time can be estimated. For our purposes it is sufficient to assume the transit time in (6) and (8) is the path length divided by the average ion velocity:

$$t \simeq b/\tilde{v} = b/\mu\tilde{E}) = b^2/(\mu V) \tag{9}$$

where $\tilde{v}$ and $\tilde{E}$ represent averaged velocity and field, respectively. With this approximation the ion density equation can be reformulated:

$$1/\rho - 1/\rho o = b^2/\epsilon V \cdot (1 + S/\rho o) \tag{10}$$

where ρ is now evaluated at the surface of the plate, and ρo at the wire. This equation assumes that the space charge offset voltage, ΔV, is a small fraction of the applied voltage, allowing the exponential term to be linearized.

One of the main consequences of these relations, (8) and (10), is that it is not possible to exchange ionic space charge for particulate space charge, even though they may be comparable in magnitude. Such an exchange may serve for conceptual purposes, but it is not accurate enough for actual calculations.

Particle Charging

The charge of individual particles depends on the local electric field and on the availability of ions. For this work, we assume an average electric field suffices to describe the environment for all the particles and that the product of ion concentration and actual exposure time is independent of the distance from wire to plate. This is consistent with earlier assumptions and is justified by the observation that the ion density is higher near the wire, but the transit time through the region of high ion density is much shorter than near the plate.

As a result, we will use the standard charging equations to estimate the charge on individual particles and then integrate over all particles to obtain the space charge. We will use the field-charging relations [2] for convenience, although other charging models apply over a wider range of particle sizes [3].

The charge on a given particle is given by

$$q(r) = qs(r)T/(1+T) \tag{11}$$

where $qs(r)$ is the saturation charge on the particle and T is the reduced ion exposure time. Here, qs, for conductive particles, is given by

$$qs(r) = 12\pi\epsilon r^2 E \tag{12}$$

where r is the particle radius and E is the field in which charging takes place. The reduced exposure time is given by

$$T = \mu\rho t/4\epsilon \tag{13}$$

where t is the real time of exposure. When we consider one wire of an ESP, t is the flow transit time through its zone of influence, and the product, ρt, will be considered fixed. When a particle passes near a corona wire, it is exposed to a high concentration of ions for a short time. When a particle passes through the charging zone near the plate, it is exposed to a lower concentration for a longer time. The reduced time, T, will be called the "age" of the particle.

For a particle distribution, the total space charge would be the integral of (13) over all particles in the volume:

$$S = \iint N(r, v)q(r)\, dr\, dv \tag{14}$$

where N is the number concentration of the particles and v is volume. The double integration can mask substantial differences in charging conditions—one reason for the simplifying assumptions above. The integral cannot be completed without a knowledge of the size distribution, so three different conceptual cases are shown in Fig. 4, using the average current density as the independent parameter. The figure is oriented in an unusual direction for later use.

For particles that enter a charging zone with a nonzero charge, the charging diagram must be modified by shifting the axes to account for the charge already acquired. This amounts to computing the age of the particles under the new charging conditions, where age means the length of time that they would have had to spend before reaching such a charge state. This is shown in Fig. 5. It is possible for particles to enter a zone with a charge that they could never have acquired in that zone, in which case their age would be greater than infinity and no charging would take place.

Particle Collection

The first step in the particle-collection evaluation is to calculate the electrical drift velocity. This is done by equating the electrical driving force with the viscous drag force to obtain a terminal velocity, w:

$$w = q(r)EC(r)/6\pi r\eta \tag{15}$$

where C is the Cunningham slip correction and η is the gas viscosity. Further details of collection depend on the collection model used: if the Deutsch–Anderson (D–A) model (see [2]) is assumed, the drift velocity will be used as the D–A migration velocity. If a turbulent diffusion model is used, then the drift velocity will be combined with a gas or particle diffusivity to obtain a concentration gradient leading to collection. The details of collection are important for the amount of material removed, but for now we only need to know that some of the space charge under the wire can be removed by collection. A schematic diagram of this removal is shown in Fig. 6, which also has an unusual orientation. The removal is shown as the space charge remaining as the particulate cloud passes under the wire.

Clearly, particle removal depends on the charge imparted to the particles and their size. If the particles reach the saturation

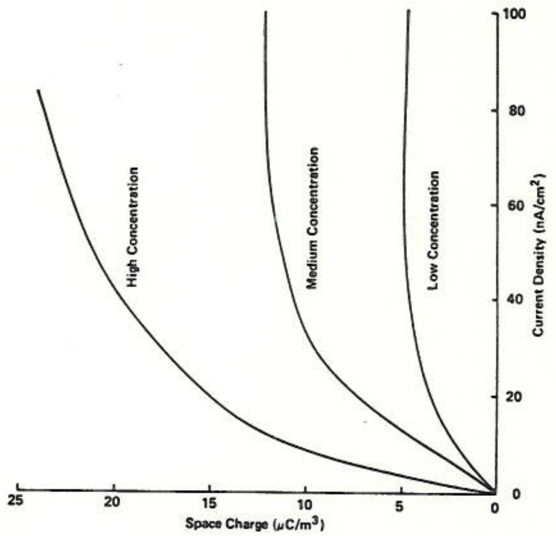

Fig. 4. Theoretical space charge at fixed voltage as function of current density for three particle distributions.

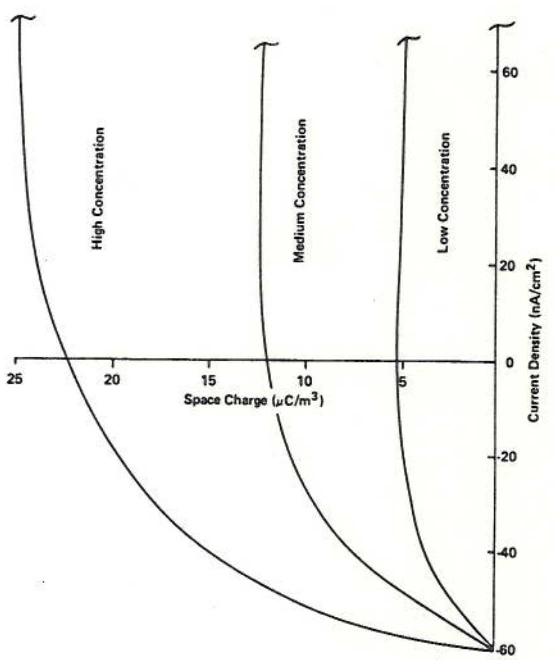

Fig. 5. Theoretical charging curves for three particle distributions with initial charge values.

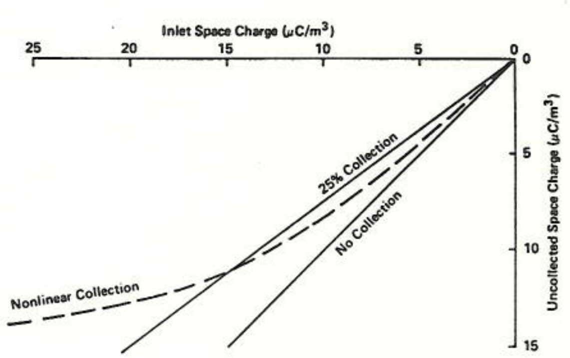

Fig. 6. Particle collection for one wire shown as removal space charge.

charge, then large-diameter particles precipitate much faster than small particles because the charge increases as the square of the diameter. This means that even a Deutsch–Anderson collection model must exhibit size-selective effects, and the removal of space charge is not likely to be linear in the amount of space charge remaining. For typical ESPs the amount removed by a single wire is a small fraction (<0.1) of the total amount, so departures from linearity are assumed to be small.

Combined Effects

The combined effects of space-charge limitation, particle charging, particle removal, and V–I curve shift are shown in Fig. 7, called an "electrostatic precipitator interaction diagram." This figure uses the individual figures presented earlier to bring the combined effects into focus.

Prior to using the diagram, some basic information should be precalculated. Starting with the V–I curves, the curve for zero space charge should be calculated (or made available for calculation by initializing its parameters). The operating voltage should be used to calculate Smax from (5). If an operating voltage is not known, [1] gives useful information, or the space charge values in Table I could be used with [1] to fix an operating voltage above the zero space-charge voltage.

From the operating voltage an average electric field should be estimated for the charging calculations. This should be a volume average from a numerical model fitted to the operating voltage, or an estimate (voltage divided by wire–plate separation), according to the desired level of accuracy. An exposure time for the particles should be estimated, for example, by dividing the average wire–wire spacing by the gas velocity, or by a more rigorous technique if desired. A particle charging theory adequate for the problem should be picked. Then, for a range of current densities from zero to a maximum practical level, the charge of particles of different sizes should be calculated, multiplied by the particle concentration, and integrated over the size distribution. This process will give a table of space charge corresponding to fixed electric field and exposure time and different levels of current density. The values in the table could be fitted with a polynomial function or interpolated at a later stage.

Using a collection theory along with the charge on the particles and the fixed electric field and exposure time, the amount of space charge removed from the gas stream can be calculated. This may result in another table, corresponding to each entry in the space charge table. Or, it may result in a linear relation between the amount of space charge entering the collection zone and the amount leaving.

The last relation is (1), giving the offset voltage as a function of the space charge near the wire. Because it is possible that the space charge is removed very effectively, it may be desirable to compute an average space charge in the direction of flow to use in (1). This could be determined by the result of the collection process.

With these precalculated quantities the diagram is ready to be used. The diagram is used by picking a point on one of the curves, for instance, corresponding to a given current density on the charging curve. From that curve the space charge is located, with the provision that it is less than the maximum

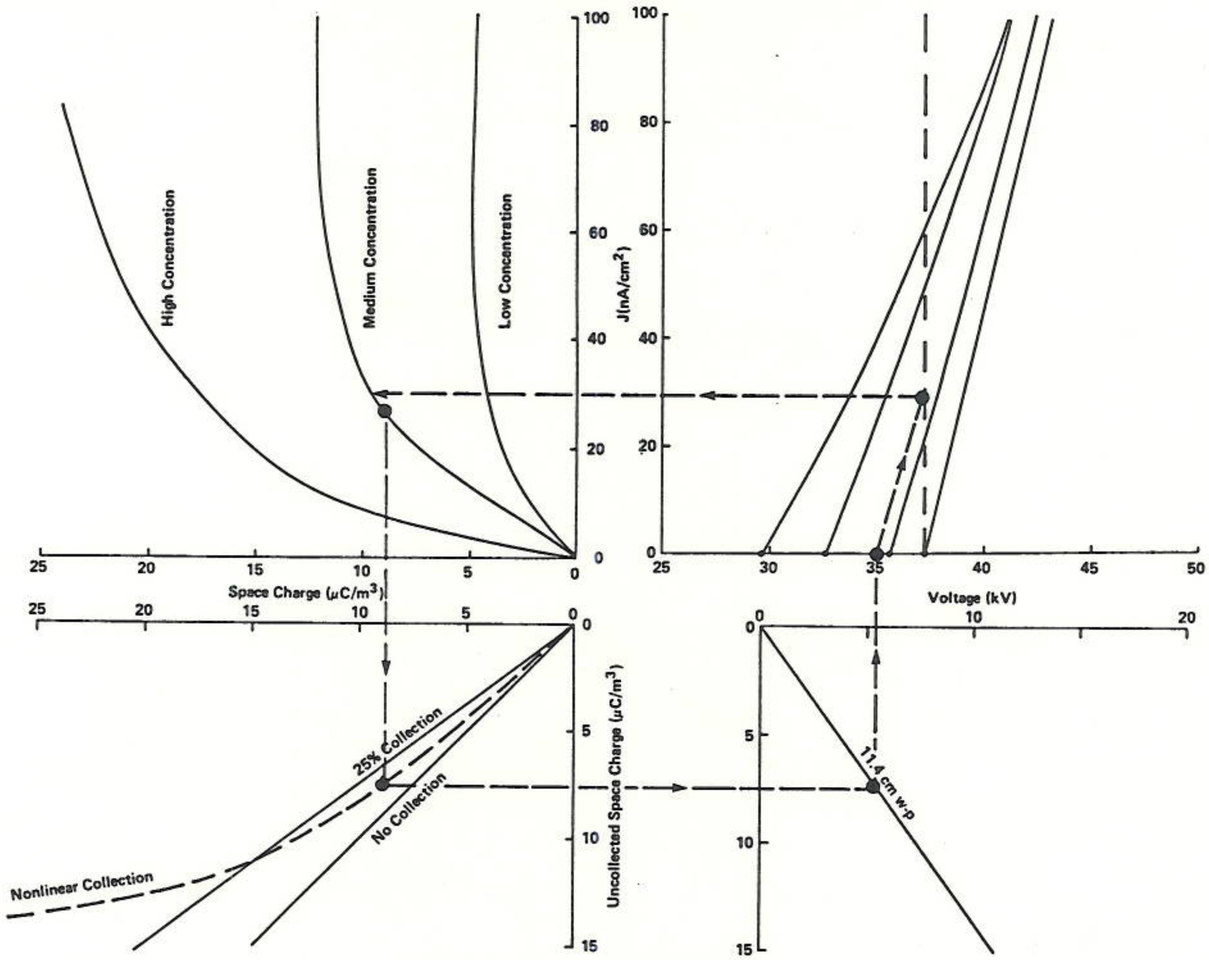

Fig. 7. Electrostatic precipitator interaction diagram. Dashed line with arrows show possible solution trajectory with small closure error.

space charge. Using that value of space charge and the collection mechanism appropriate for the particle size distribution, the space charge remaining after collection is obtained. From that, the voltage offset caused by space charge is obtained. Using the space-charge offset, the appropriate V–j curve is followed up to the operating voltage to obtain the actual current density.

The square trajectory around the diagram will not usually close on itself, and a sampling of such trajectories shows that there are many situations in which the trajectories lead to either maximum or minimum space charge values. Numerical damping techniques would allow a fairly rapid approach to a self-consistent trajectory. The best starting point is usually slightly below the maximum space charge value. The particle-charging curve may set a limit on the allowable current density to reach that value of space charge. For instance, at high concentrations, only low current densities would be allowed within the maximum space-charge limit. It is essential for this process that the operating voltage be fixed. This keeps the average electric field constant in the charging calculations and makes the space charge a function of current density only.

An alternate process is to solve for the appropriate current density graphically, by making use of a compound curve called the "specific voltage offset curve." This curve is a combination of the charging, collection, and offset quadrants of Fig. 7 and the space-charge-dependent slope of Fig. 3, plotted as a current-density-dependent voltage. It produces a plot similar to a load-line diagram for a transistor or vacuum tube, shown in Fig. 8. The intersection of the specific voltage offset curve with the clean plate V–j curve is the operating current density for the system.

This offset curve's equation is given by

$$Vo(j)=\iint N(r, v)q(r, j)\, dr\, dv/(2\epsilon b^2)\cdot(1-Fc)+j/[\Delta K\cdot V\cdot(V-VO)] \quad (16)$$

where Fc is the collected fraction, and the last term represents the change in voltage caused by the change in slope of the space charge V–j curves.

Either of these approaches can be used in progressing from one wire to another through the ESP. The particle–charging curve will be shifted for successive wires (as in Fig. 5) and the number of particles will be reduced by the collection process; the application of the techniques remains the same. After the space charge is reduced below 1 $\mu C/m^3$, it will have little effect on the V–J curve and probably can be neglected. Similarly, the particles will cease charging at some point, and the charging calculations can be neglected.

In recent work [7] this approach has been implemented in another way. The voltage of an entire electrical section is

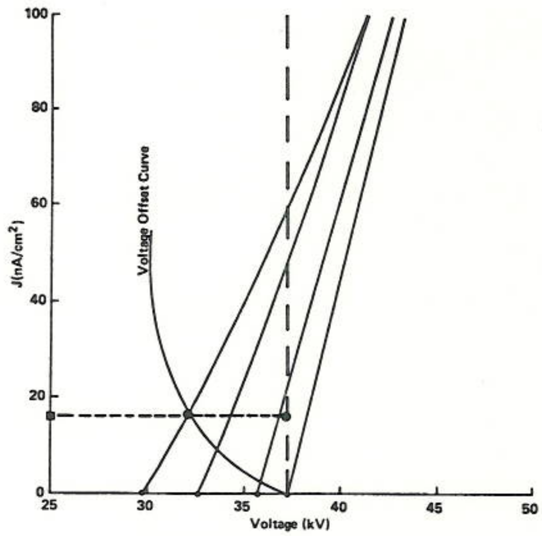

Fig. 8. Solution of charging problem using "specific voltage offset curve."

fixed, and the space charge is assumed to decay exponentially with distance along the section. Under these conditions, average current density and electric fields are calculated. These average values are then used to calculate particulate space charge using a combined field and diffusion-charging theory; at the same time, the particles are collected along the length of the section. The space charges calculated at the inlet and outlet of the section are used to compute an exponential decay rate for the space charge. The inlet space charge and decay rate, then, serve as inputs for the next iteration of the current-density field calculations. The process converges rapidly at normal particulate loadings and seems adequate for computing the electrical operation of most ESPs.

Conclusion

The procedure described here is a formal attempt to solve the complex interactions occurring in the charging zones of an ESP. Although many of the details cannot be completed without reference to specific particle size distributions and choice of particle-charging and collection theories, the overall process allows a rapid approximate value of the operating current density for a given voltage. The results are dependent on the predicted behavior of the voltage current characteristics in the presence of space charge and, invariably, refer back to the clean-plate characteristics.

Details of the charging mechanisms, even at low current densities, have been glossed over. Similarly, the collection process has been treated lightly by assuming that a rather small fraction of the space charge is removed during the charging process. Any of these assumptions can be refined with some effort, but the principal conclusion remains: the particulate space charge is the primary determinant of the operating point.

However much effort is put into detailed charging and collection models, the inescapable conclusion is that the electrical conditions respond to an average space charge. The calculation of that space charge should become the focus for ESP charging theories.

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Philip A. Lawless for a biography and photograph, please see page 939 of this TRANSACTIONS.

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