

# A COMPARISON OF NUMERICAL AND ANALYTICAL RADIATIVE-TRANSFER SOLUTIONS FOR PLANE ALBEDO IN NATURAL WATERS

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Several numerical and analytical solutions of the radiative transfer equation were compared for plane albedo in a problem of solar light reflection by sea water.

Several numerical and analytical solutions of the radiative transfer equation (RTE) for plane albedo were compared for solar light reflection by sea water. The study incorporated the simplest case, that being a semi-infinite one-dimensional plane-parallel absorbing and scattering homogeneous layer illuminated by a monodirectional light beam. Inelastic processes (i.e., Raman scattering and fluorescence), polarization and air-water surface refraction-reflection effects, were not considered. Algorithms were based on the invariant imbedding method and two different variants of the discrete ordinate method (DOM). Calculations were performed using parameters across all possible ranges including single-scattering albedo  $\omega_0$  and refracted solar zenith angle  $\theta_1$ , but with a special emphasis on natural waters. All computations were made for two scattering phase functions, which included an almost isotropic Rayleigh phase function and strongly anisotropic double-peaked Fournier-Forand-Mobley phase function. Models were validated using quasi-single-scattering (QSSA) and exponential approximations to represent the extreme cases of  $\omega_0 \rightarrow 0$  and  $\omega_0 \rightarrow 1$ , respectively. All methods yielded relative differences within 1.8% for modeled natural waters. An analysis of plane albedo behavior resulted in the development of a new extended QSSA approximation, which when applied in conjunction with the extended Hapke approximation developed earlier, resulted in a maximum relative error of 2.7%. The study results demonstrated that for practical applications, the estimation of inherent optical properties from observed reflectance can best be achieved using an extended Hapke approximation.

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